MTH 131: Mathematical Analysis for Management, Fall 2017

Practice Midterm 3

Name: ______________________________

Student Number: ______________________________

Answer the questions in the spaces provided on the question sheets.

Show all of your work.

If you run out of room for an answer, continue on the back of the page.

You are allowed to use a non-graphing calculator.

You are allowed a one page formula sheet.

Page	Points	Score
3	10	
4	6	
5	8	
6	2	
7	8	
8	11	
9	10	
10	11	
11	9	
12	8	
13	9	
14	8	
Total:	100	

1. Evaluate the following indefinite integral: $\int 6x\,dx$ [2]

$\int 6x\,dx =$ ______________________ (Use C as the arbitrary constant.)

2. Find the following indefinite integral: $\int \frac{1}{x^6}\,dx$ [2]

$\int x^{-6}\,dx =$ ______________________ (Use C as the arbitrary constant.)

3. Evaluate: $\int \frac{8}{x}\,dx$ [2]

$\int \frac{8}{x}\,dx =$ ______________________ (Use C as the arbitrary constant.)

4. Discuss the validity of the following statement, and select the correct answer below. [1]

If n is an integer, then $\dfrac{x^{n+1}}{n+1}$ is an antiderivative of x^n.

- ◯ This is a true statement for all n.
- ◯ This is a false statement for all n.
- ◯ This is a true statement for all n except -1.
- ◯ This is a true statement for all positive n.

5. Evaluate the following indefinite integral: $\int \frac{3}{\sqrt{x}}\,dx$ [3]

$\int \frac{3}{\sqrt{x}}\,dx =$ ______________________ (Use C as the arbitrary constant.)

6. Find the particular antiderivative of the following derivative that satisfies the given condition. [3]

$$\frac{dx}{dt} = 6e^t - 5; \qquad x(0) = 2$$

$x(t) =$ ________________________

7. Find the indefinite integral: $\displaystyle\int \frac{x^3}{4+5x^4}\,dx$ [3]

$\displaystyle\int \frac{x^3}{4+5x^4}\,dx =$ ________________________ (Use C as the arbitrary constant.)

8. Find the indefinite integral: $\int x\sqrt{7-x^2}\,dx$ [3]

$\int x\sqrt{7-x^2}\,dx =$ ______________________________ (Use C as the arbitrary constant.)

9. Find the general solution for the first-order differential equation: $\dfrac{dy}{dx} = 28x.$ [2]

$y =$ ______________________________ (Use C as the arbitrary constant.)

10. Find the particular solution to the first-order differential equation that satisfies the given condition: [3]

$$\frac{dy}{dx} = e^{0.5x}; \qquad y(0) = 3$$

$y =$ ______________________________

11. Give the order of the differential equation below, where y represents a function of x. [1]

$$y + x^4 y'' - 2y' = 2$$

◯ The given equation is a first-order differential equation because it only involves a first derivative.

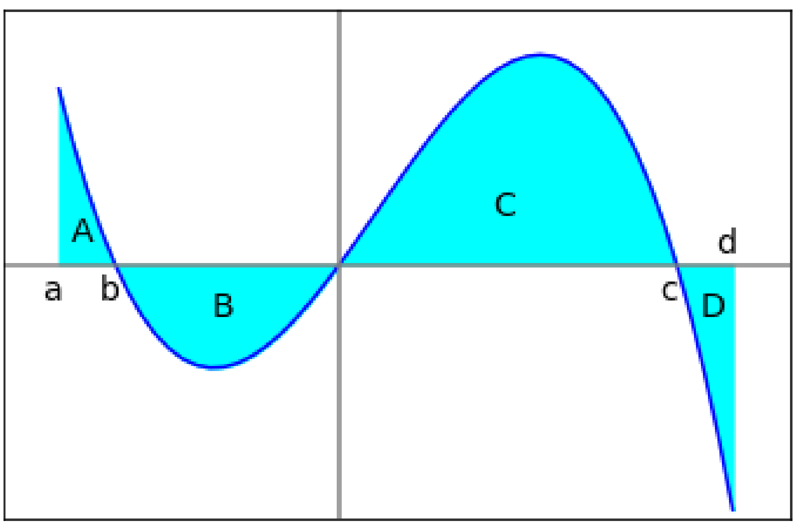
◯ The given equation is a first-order differential equation because it involves a first derivative but no higher derivative.

◯ The given equation is a second-order differential equation because it involves a second derivative but no higher derivative.

◯ The given equation is a third-order differential equation because it involves a third derivative but no higher derivative.

12. Calculate the definite integral below by referring to the figure with the indicated areas. [1]

$$\int_0^c f(x)\,dx$$



Area A = 1.213   Area B = 2.71

Area C = 5.369   Area D = 1.611

$\int_0^c f(x)\,dx =$ ____________________ (Write the numerical value.)

13. Evaluate the integral: $\int_3^4 (2x+3)\,dx$. [3]

$$\int_3^4 (2x+3)\,dx = \underline{\hspace{6cm}}$$

14. Calculate the definite integral: $\int_3^6 \frac{7}{x}\,dx$. [3]

$$\int_3^6 \frac{7}{x}\,dx = \underline{\hspace{6cm}}$$

15. Evaluate the integral: $\int_4^4 \left(x^2-6x+9\right)^{15}\,dx$ [2]

$$\int_4^4 \left(x^2-6x+9\right)^{15}\,dx = \underline{\hspace{6cm}}$$

16. Calculate the definite integral: $\int_0^1 5\sqrt[5]{x}\,dx$

$$\int_0^1 5\sqrt[5]{x}\,dx = \underline{\hspace{6cm}}$$

17. Find the area bounded by the graphs of the equations $f(x)$ and $g(x)$ over the given interval. [4]

$$f(x) = x^2 - 6; \quad g(x) = 3; \quad 0 \leq x \leq 3$$

The area is ________________________ square units.

18. Find the area bounded by the graphs of the equations $f(x)$ and $g(x)$. [4]

$$f(x) = 4x^2; \quad g(x) = 36$$

The area, calculated to three decimal places, is ________________________ square units.

19. Find the Gini index of income concentration for the Lorenz curve given by: $f(x) = x^{2.4}$. [3]

The Gini index is ________________________ (Round to three decimal places.)

20. Find the total income produced by a continuous income stream in the first 3 years if the rate of flow is given by the following function, where t is time in years: [3]

$$f(t) = 400e^{0.05t}$$

The total income is ________________________ (Round to the nearest dollar.)

21. Find the future value at 3.25% interest, compounded continuously for 4 years, of the continuous income stream with rate of flow: [4]

$$f(t) = 1650e^{-0.02t}$$

The future value is ________________________ (Round to the nearest dollar.)

22. Find the consumers' surplus at a price level of $\bar{p} = \$120$ for the price-demand equation below: [3]

$$p = D(x) = 400 - 0.02x$$

The consumers' surplus is ________________________ (Round to the nearest dollar.)

23. Find the producers' surplus at a price level of $\bar{p} = \$67$ for the price-supply equation below: [4]

$$p = S(x) = 10 + 0.1x + 0.0003x^2$$

The producers' surplus is ________________________ (Round to the nearest dollar.)

24. Evaluate the integral: $\displaystyle\int xe^{6x}\,dx$ [6]

$\displaystyle\int xe^{6x}\,dx-$ ________________________

25. A company produces two models of a surfboard: a standard model and a competition model. The monthly cost function is given by: [1]

$$C(x, y) = 3000 + 150x + 310y$$

where x is the number of standard models produced per month and y is the number of competition models produced per month. Find $C(16, 8)$.

$C(16, 8) =$ ________________________

26. The Cobb-Douglas production function for a bicycle company is given by: [2]

$$f(x, y) = 19x^{0.7}y^{0.3}$$

where x is the utilization of labor and y is the utilization of capital. If the company uses 1,211 units of labor and 1,742 units of capital, how many bicycle will be produced?

The company will produce ________________ bicycles (Round to the nearest integer.)

27. Find $\dfrac{\partial z}{\partial y}$ if $z = x^2 - 4xy + 5y^2$ [3]

$\dfrac{\partial z}{\partial y} =$ ______________________

28. Find $f_{xx}(x, y)$ if $f(x, y) = (5x + y)^5$ [4]

$f_{xx}(x, y) =$ ______________________

29. Find the critical points of the function:

$$f(x, y) = x^2 - y^2 + 4x + 6y - 8$$

Use the second derivative test to determine if the critical points are local extrema.

(a) Find the critical points of $f(x, y)$. Use a comma to separate answers as needed. [4]

The function $f(x, y)$ has critical points at ____________________

(b) Use the second derivative test to determine if the critical points are local extrema. [1]

Find the location of local minima.

○ The function has local minima at ____________________

○ The function has no local minima.

(c) Find the location of local maxima. [1]

○ The function has local maxima at ____________________

○ The function has no local maxima.

(d) Find the location of any saddle points. [1]

○ The function has saddle points at ____________________

○ The function has no saddle points.

(e) Find the location of any critical points where the second-derivative test fails. [1]

○ The second-derivative test fails at ____________________

○ The second derivative test does not fail for any of the critical points.

30. Explain why $f(x, y) = x^2$ has local extremum at infinitely many points. [1]

- ◯ $f_x(x, y)$ and $f_y(x, y)$ are always negative except at $x = 0$, where $f(x, y) = 0$, so f has the local maximum 0 at each point of the y-axis.
- ◯ $f_x(x, y)$ and $f_y(x, y)$ are both equal to zero for $x = 0$ and y is any real number, meaning that each of these critical points represents a critical point. Since $f(x, y)$ is nonnegative and equals zero when $x = 0$, f has the local minimum 0 at each point of the y-axis.
- ◯ $f_x(x, y)$ and $f_y(x, y)$ are both equal to zero for all points on the y-axis. Since $AC-B^2 < 0$ and $A < 0$ for all points on the x-axis, where $A = f_{xx}(a, b)$, $B = f_{xy}(a, b)$, $c = f_{yy}(a, b)$, and (a, b) represents a critical point, $f(x, y)$ has local minima for all points on the x-axis.
- ◯ $f_x(x, y)$ and $f_y(x, y)$ are always negativeexcept at $x = 0$, where $f(x, y) = 0$, so f has the local maximum 0 at each point of the x-axis.

31. Use the method of Lagrange Multipliers to maximize $f(x, y) = 2xy$ subject to $x + y = 6$ [8]

The maximum value of $f(x, y)$ is ____________ at $x =$ ____________, $y =$ ____________

32. Three pens of the same size are to be built along an existing fence. Let the fence parallel to the existing fence be of length x, and the length of the other four fences each be y. If 640 feet of fencing are available, what length should x and y be to produce the maximum total area? What is the maximum area? Use any technique you wish to find the maximum area. [8]

The maximum area is ____________ square feet when $x =$ _________ feet, $y =$ _________ feet.