MTH 131: Mathematical Analysis for Management, Fall 2017

Practice Midterm 2

Name: ______________________________

Student Number: ______________________________

Answer the questions in the spaces provided on the question sheets.

Show all of your work.

If you run out of room for an answer, continue on the back of the page.

You are allowed to use a non-graphing calculator.

You are allowed a one page formula sheet.

Page	Points	Score
3	7	
4	9	
5	7	
6	8	
7	8	
8	7	
9	8	
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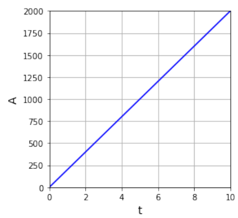
13	6	
14	6	
15	6	
16	2	
Total:	100	

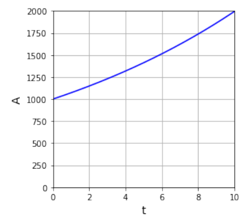
1. If \$1000 is invested at 6.9% compounded continuously, graph the amount in the account as a function of time for a period of 9 years. [1]

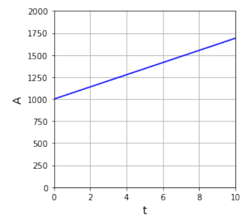
   Choose the correct graph.

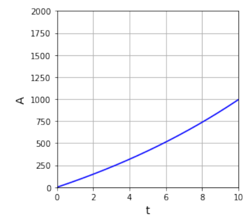
   Full solution:

   Choose the graph with a y-intercept of \$1000 that shows exponential growth.

○ 

√ 

○ 

○ 

2. Recently, a certain bank offered a 10-year CD that earns 10.19% compounded continuously.

   (a) If \$30,000 is invested in this CD, how much will it be worth in 10 years? [3]

   Full solution:

$$\begin{aligned} A &= Pe^{rt} && P = 30,000, r = 0.1019, t = 10 \\ &= 30000e^{0.1019(10)} \\ &= 83112.69 \end{aligned}$$

   Approximately \$ **83,112.69** (Round to the nearest cent)

   (b) How long will it take for the account to be worth \$75,000? [3]

   Full solution:

$$\begin{aligned} A &= Pe^{rt} && A = 75,000, P = 30,000, r = 0.1019 \\ 75000 &= 30000e^{0.1019t} \\ \frac{75000}{30000} &= e^{0.1019t} \\ 2.5 &= e^{0.1019t} \\ \ln(2.5) &= 0.1019t \\ t &= \ln(2.5)/0.1019 \\ &= 8.99 \end{aligned}$$

   Approximately **8.99** years (Round to two decimal places as needed).

3. Find $f'(x)$ for $f(x) = 7e^x + 3x - \ln x$. [3]

Full solution:
$f'(x) = \dfrac{d}{dx}(7e^x + 3x - \ln x) = 7\dfrac{d}{dx}e^x + 3\dfrac{d}{dx}x - \dfrac{d}{dx}\ln x = 7e^x + 3 - \dfrac{1}{x}$

$f'(x) =$ $\underline{\qquad 7e^x + 3 - 1/x \qquad}$

4. Find $\dfrac{dy}{dx}$ for $y = 2\log_9 x$. [2]

Full solution:
$\dfrac{d}{dx}\log_b x = \dfrac{1}{x \ln b}$, so $\dfrac{d}{dx} 2\log_9 x = 2\dfrac{1}{x\ln 9} = \dfrac{2}{x \ln 9} \left(= \dfrac{2}{x \ln 3^2} = \dfrac{2}{2x \ln 3} = \dfrac{1}{x \ln 3}\right)$

$\dfrac{dy}{dx} =$ $\underline{\qquad 2/(x \ln 9) \qquad}$

5. Find $\dfrac{dy}{dx}$ for the function $y = 2^x$. [2]

Full solution:
$\dfrac{d}{dx}b^x = b^x \ln b$, so $\dfrac{d}{dx}2^x = 2^x \ln 2$

$\dfrac{dy}{dx} =$ $\underline{\qquad 2^x \ln 2 \qquad}$

6. Find $f'(x)$ for $f(x) = \dfrac{x}{x+18}$. [2]

Full solution:

By quotient rule,

$$\frac{d}{dx}\frac{x}{x+18} = \frac{(x+18)\frac{d}{dx}(x) - x\frac{d}{dx}(x+18)}{(x+18)^2} = \frac{(x+18)(1) - x(1)}{(x+18)^2} = \frac{18}{(x+18)^2}$$

$f'(x) =$ $\underline{\qquad 18/(x+18)^2 \qquad}$

7. Find $f'(x)$ for $f(x) = 18xe^x$. [2]

Full solution:

By product rule,

$$\frac{d}{dx}(18x)(e^x) = 18x\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(18x) = 18x(e^x) + e^x(18)$$

$f'(x) =$ $\underline{\qquad 18xe^x + 18e^x \qquad}$

8. Find $f'(x)$ for $f(x) = 5x^4 \ln x$. [2]

Full solution:

By product rule,

$$\frac{d}{dx}(5x^4)(\ln x) = 5x^4\frac{d}{dx}$$

$f'(x) =$ $\underline{\qquad 5x^3 + 20x^3 \ln x \qquad}$

9. Use the quotient rule to find the derivative of $y = \dfrac{9x^2+7}{x^2+1}$ [3]

Full solution:

By quotient rule,

$$\begin{aligned}\frac{d}{dx}\frac{9x^2+7}{x^2+1} &= \frac{(x^2+1)\frac{d}{dx}(9x^2+7) - (9x^2+7)\frac{d}{dx}(x^2+1)}{(x^2+1)^2} \\ &= \frac{(x^2+1)(18x) - (9x^2+7)(2x)}{(x^2+1)^2} \\ &= \frac{(18x^3+18x) - (18x^3+14x)}{(x^2+1)^2} \\ &= \frac{4x}{(x^2+1)^2}\end{aligned}$$

$f'(x) =$ $\underline{\qquad 4x/(x^2+1)^2 \qquad}$

10. Find $f'(x)$ for $f(x) = (3-5x)^{15}$. [2]

Full solution:

By general power rule, $\frac{d}{dx}[u(x)]^n = n[u(x)]^{n-1}u'(x)$. Let $u(x) = 3-5x$, so:

$$\frac{d}{dx}(3-5x)^{15} = 15[3-5x]^{14}(-5) = -75(3-5x)^{14}$$

$f'(x) =$ $\underline{\quad -75(3-5x)^{14} \quad}$

11. Find $f'(x)$ for $f(x) = 9\ln(5x^2+6)$. [2]

Full solution:

By chain rule, $\frac{d}{dx}\ln[f(x)] = \frac{1}{f(x)}f'(x)$. Let $f(x) = 5x^2+6$, so:

$$\frac{d}{dx}9\ln(5x^2+6) = 9\frac{1}{5x^2+6}\frac{d}{dx}(5x^2+6) = \frac{9}{5x^2+6}(10x) = \frac{90x}{5x^2+6}$$

$f'(x) =$ $\underline{\quad 90x/(5x^2+6) \quad}$

12. For $f(x) = 4e^{x^2-5x+7}$:

(a) Find $f'(x)$. [2]

Full solution:

By chain rule, $\frac{d}{dx}e^{f(x)} = e^{f(x)}f'(x)$. Let $f(x) = x^2-5x+7$, so

$$\frac{d}{dx}4e^{x^2-5x+7} = 4e^{x^2-5x+7}\frac{d}{dx}(x^2-5x+7) = 4e^{x^2-5x+7}(2x-5) = 4(2x-5)e^{x^2-5x+7}$$

$f'(x) =$ $\underline{\quad 4(2x-5)e^{x^2-5x+7} \quad}$

(b) Find the equation of the tangent line to the graph of f when $x = 0$ [2]

Full solution:

When $x = 0$, $f(0) = 4e^{0^2-5(0)+7} = 4e^7$, and $f'(0) = 4(2(0)-5)e^{0^2-5(0)+7} = -20e^7$. Plugging into the slope-intercept form $y = mx+b$ with $m = -20e^7$, $b = 4e^7$, yields

$$y = -20e^7x + 4e^7$$

$f'(x) =$ $\underline{\quad y = -20e^7x + 4e^7 \quad}$

(c) Find the value(s) of x where the tangent line is horizontal. [2]

Full solution:

The tangent line is horizontal when $f'(x) = 0$, so $4(2x - 5)e^{x^2-5x+7} = 0$, Since $e^{x^2-5x+7} > 0$ for all x, this means $2x - 5 = 0$, so $x = 5/2$.

√ **The tangent line is horizontal at x = 5/2**

◯ The tangent line is never horizontal.

13. If it is possible to solve for y in terms of x, do so: $3x - 4y = e^y$. [2]

Select the correct choice below.

Full solution:

Can't be done. Try it.

◯ y = ____________

√ **It is impossible to solve the equation for y in terms of x.**

14. For the equation $4x^3 - y^2 - 7 = 0$:

(a) Use implicit differentiation to find y'. [3]

Full solution:

Differentiate each term with respect to x, using the chain rule for y^2:

$$\frac{d}{dx}(4x^3 - y^2 - 7) = \frac{d}{dx}(4x^3) - \frac{d}{dy}(y^2)\frac{dy}{dx} - \frac{d}{dx}7 = 12x^2 - 2yy' = 0$$

Solve for y':

$$y' = \frac{12x^2}{2y} = \frac{6x^2}{y}$$

$y' =$ $6x^2/y$

(b) Evaluate y' at (2, 5). [1]

Full solution:

$$y' = \frac{6x^2}{y} = \frac{6(2^2)}{5} = \frac{24}{5}$$

$y'(2,5) =$ **24/5**

15. For the equation $y^2 + 3y + 4x = 0$:

(a) Use implicit differentiation to find y'. [3]

Full solution:

Differentiate each term with respect to x, using the chain rule for y^2:

$$\frac{d}{dx}(y^2 + 3y + 4x) = \frac{d}{dy}(y^2)\frac{dy}{dx} + 3\frac{dy}{dx} + 4\frac{d}{dx}x = 2yy' + 3y' + 4 = 0$$

Solve for y':

$$2yy' + 3y' + 4 = 0 \Rightarrow (2y+3)y' = -4 \Rightarrow y' = \frac{-4}{2y+3}$$

$y' =$ $-4/(2y+3)$

(b) Evaluate y' at (-7, 4). [1]

Full solution:

$$\frac{-4}{2y+3} = \frac{-4}{2(4)+3} = -\frac{4}{11}$$

$y'(2,5) =$ **-4/11**

16. Assume that $x = x(t)$ and $y = y(t)$. Let $y = x^3 + 4$ and $\dfrac{dx}{dt} = 4$ when $x = 3$. [3]

Find $\dfrac{dy}{dt}$ when $x = 3$.

Full solution:

Differentiate each term with respect to t, using the chain rule for x^3:

$$\frac{dy}{dt} = \frac{d}{dx}(x^3)\frac{dx}{dt} + \frac{d}{dt}4 = 3x^2\frac{dx}{dt}$$

Evaluate dy/dt when $dx/dt = 4$ and $x = 3$:

$$\frac{dy}{dt} = 3x^2\frac{dx}{dt} = 3(3^2)(4) = 108$$

$\dfrac{dy}{dt} =$ **108**

17. Assume that $x = x(t)$ and $y = y(t)$. Find $\dfrac{dx}{dt}$ using the following information: [3]

$x^2 + y^2 = 585$; $\dfrac{dy}{dt} = -4$ when $x = -21$ and $y = 12$.

Full solution:

Differentiate each term with respect to t, using the chain rule for x^2 and y^2:

$$\frac{d}{dx}(x^2)\frac{dx}{dt} + \frac{d}{dy}(y^2)\frac{dy}{dt} = 0 \Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Find dx/dt when $dy/dt = -4$, $x = -21$, and $y = 12$:

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow 2(-21)\frac{dx}{dt} + 2(12)(-4) = 0 \Rightarrow \frac{dx}{dt} = -\frac{16}{7}$$

$\dfrac{dx}{dt} =$ **-16/7** (Type an integer or a simplified fraction)

18. A point is moving on the graph of $xy = 12$. When the point is at (4, 3), its x-coordinate is increasing by 4 units per second. How fast is the y-coordinate changing at that moment? [3]

Full solution:

Differentiate the expression $xy = 12$ with respect to t, using the product rule:

$$\frac{d}{dt}(xy) = x\frac{dy}{dt} + y\frac{dx}{dt} = 0$$

Plug in the known values $dx/dt = 4$ when $x = 4$ and $y = 3$, and solve for dy/dt:

$$x\frac{dy}{dt} + y\frac{dx}{dt} = (4)\frac{dy}{dt} + (3)(4) = 0 \Rightarrow \frac{dy}{dt} = -3$$

The y-coordinate is ◯ increasing √ **decreasing** at **3** units per second.

19. Find the relative rate of change of $f(x) = 15 + 3e^{-3x}$. [2]

Full solution:

The relative rate of change of $f(x)$ is $f'(x)/f(x)$. By the chain rule,

$$\frac{d}{dx}e^{-3x} = e^{-3x}\frac{d}{dx}(-3x) = -3e^{-3x} \Rightarrow \frac{f'(x)}{f(x)} = \frac{3(-3e^{-3x})}{15 + 3e^{-3x}} = \frac{-9e^{-3x}}{15 + 3e^{-3x}}$$

The relative rate of change is $-9e^{-3x}/(15 + 3e^{-3x})$

20. For $f(x) = 126 + 32x$:

(a) Find the percentage rate of change of $f(x)$. [2]

Full solution:

The percentage rate of change of $f(x)$ is

$$100f'(x)/f(x) = 100\frac{32}{126+32x} = \frac{3200}{126+32x}$$

The percentage rate of change is **3200/(126 + 32x)**

(b) Evaluate the percentage rate of change of $f(x)$ when $x = 6$. Round your answer to 1 decimal place. [1]

Full solution:

$$\frac{3200}{126+32x} = \frac{3200}{126+32(6)} = \frac{3200}{318} = 10.1\%$$

The percentage rate of change when $x = 6$ is **10.1** %.

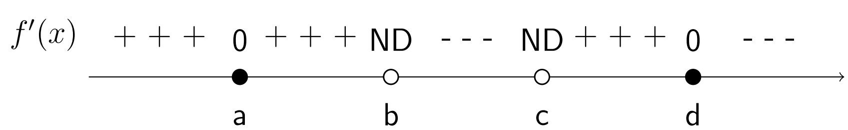
21. Use the price-demand equation $x = f(p) = 30,000 - 700p$ to find $E(p)$, the elasticity of demand. [3]

Full solution:

$$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{p(-700)}{30000-700p} = \frac{7p}{300-7p}$$

$E(p) =$ **7p/(300 - 7p)**

22. $f(x)$ is continuous on $(-\infty, \infty)$ and has critical numbers at $x = a$, b, c, and d. Use the sign chart below for $f'(x)$ to determine whether f has a local maximum, a local minimum, or neither at each critical number.



(a) Does $f(x)$ have a local minimum, a local maximum, or no local extremum at $x = a$? [1]

Full solution:

$f'(x)$ does not change sign at $x = a$, so this is not a local extremum.

◯ a local minimum ◯ a local maximum √ **no local extremum**

(b) Does $f(x)$ have a local minimum, a local maximum, or no local extremum at $x = b$? [1]

Full solution:

$f(x)$ changes from increasing to decreasing at $x = b$, so this is a local maximum.

◯ a local minimum $\surd$ **a local maximum** ◯ no local extremum

(c) Does $f(x)$ have a local minimum, a local maximum, or no local extremum at $x = c$? [1]

Full solution:

$f(x)$ changes from decreasing to increasing at $x = c$, so this is a local minimum.

$\surd$ **a local minimum** ◯ a local maximum ◯ no local extremum

(d) Does $f(x)$ have a local minimum, a local maximum, or no local extremum at $x = d$? [1]

Full solution:

$f(x)$ changes from increasing to decreasing at $x = d$, so this is a local maximum.

◯ a local minimum $\surd$ **a local maximum** ◯ no local extremum

23. Find the intervals on which $f(x)$ is increasing, the intervals on which $f(x)$ is decreasing, and the local extrema for $f(x) = -5x^2 - 30x - 18$.

Type your answers using interval notation, and use a comma to separate answers as needed.

(a) Where is $f(x)$ increasing? [3]

Full solution:

The function is increasing when $f'(x) > 0$. Since $f'(x) = -10x - 30 = -10(x + 3)$, then:

$$f'(x) > 0 \Rightarrow -10(x + 3) > 0 \Rightarrow x + 3 < 0 \Rightarrow x < -3$$

$\surd$ **The function is increasing on** **($-\infty$, -3)**

◯ There is no solution.

(b) Where is $f(x)$ decreasing? [3]

Full solution:

The function is decreasing when $f'(x) < 0$:

$$f'(x) < 0 \Rightarrow -10(x + 3) < 0 \Rightarrow x + 3 > 0 \Rightarrow x > -3$$

$\surd$ **The function is decreasing on** **(-3, ∞)**

◯ There is no solution.

(c) Which statement is true regarding the local extrema? [2]

Full solution:
The function changes increasing $\to$ decreasing at $x = -3$, so this is a local maximum.

◯ The function has a local minimum at $x =$ ________________

√ **The function has a local maximum at** $x =$ **-3**

◯ The function has no local extrema.

24. Find $f''(x)$ for $f(x) = 2x^4 - 3x^3 + 3x - 9$. [2]

Full solution:

$$f(x) = 2x^4 - 3x^3 + 3x - 9 \quad \Rightarrow f'(x) = 8x^3 - 9x^2 + 3 \quad \Rightarrow f''(x) = 24x^2 - 18x$$

$f''(x) =$ $24x^2 - 18x$

25. For the function $f(x) = x^{18} + 3x^2$, find the intervals on which the graph of f is concave upward, the intervals on which the graph of f is concave downward, and the inflection points.

(a) For what interval(s) of x is the graph of f concave upward? [2]

Full solution:
Concavity depends on the sign of the second derivative:

$$f(x) = x^{18} + 3x^2 \quad \Rightarrow f'(x) = 18x^{17} + 6x \quad \Rightarrow f''(x) = 306x^{16} + 6 > 0 \text{ for all } x$$

So, the graph of f is concave upward for all x.

√ **The graph is concave upward on the interval(s)** **$(-\infty, \infty)$**

◯ The graph is never concave upward.

(b) For what interval(s) of x is the graph of f concave downward? [2]

Full solution:
$f''(x) > 0$ for all x, so the graph of f is never concave downward.

◯ The graph is concave downward on the interval(s) ________________

√ **The graph is never concave downward.**

(c) Determine the x-coordinates of any inflection points of the graph of $f(x)$. Use a comma to separate your answers. [2]

Full solution:
$f''(x) > 0$ everywhere, so there are no inflection points.

◯ There are inflection points at $x =$ ________________

√ **There are no inflection points.**

26. Use L'Hôpital's rule to find the limit $\lim_{x\to 0} \frac{e^{2x}-1}{3x}$. Use $-\infty$ and ∞ when appropriate. [3]

Full solution:

$$\lim_{x\to 0} e^{2x} - 1 = e^{2(0)} - 1 = 0 \quad \text{and} \quad \lim_{x\to 0} 3x = 3(0) = 0$$

So, L'Hôpital's rule applies:

$$\lim_{x\to 0} \frac{e^{2x}-1}{3x} \overset{\text{L'Hôp}}{=} \lim_{x\to 0} \frac{\frac{d}{dx}(e^{2x}-1)}{\frac{d}{dx}(3x)} = \lim_{x\to 0} \frac{2e^{2x}}{3} = \frac{2e^0}{3} = \frac{2}{3}$$

√ $\lim_{x\to 0} \frac{e^{2x}-1}{3x} =$ **2/3**

◯ The limit does not exist.

27. Use L'Hôpital's rule to find the limit $\lim_{x\to 0} \frac{\ln(1+2x)}{x}$. Use $-\infty$ and ∞ when appropriate. [3]

Full solution:

$$\lim_{x\to 0} \ln(1+2x) = \ln(1+2(0)) = 0 \quad \text{and} \quad \lim_{x\to 0} x = 0$$

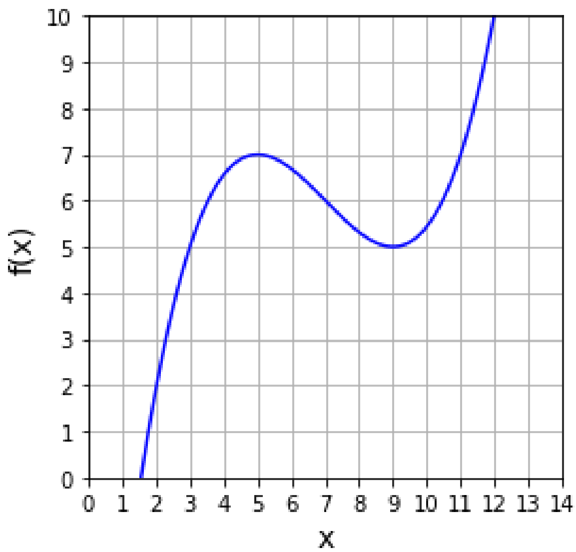
So, L'Hôpital's rule applies. Use the chain rule to find the derivative of $\ln(1+2x)$:

$$\lim_{x\to 0} \frac{\ln(1+2x)}{x} \overset{\text{L'Hôp}}{=} \lim_{x\to 0} \frac{\frac{d}{dx}\ln(1+2x)}{\frac{d}{dx}(x)} = \lim_{x\to 0} \frac{\frac{2}{1+2x}}{1} = \frac{\frac{2}{1+2(0)}}{1} = 2$$

√ $\lim_{x\to 0} \frac{\ln(1+2x)}{x} =$ **2**

◯ The limit does not exist.

28. Refer to the graph shown below. Find the absolute minimum and the absolute maximum over the interval [2, 10]. Round to the nearest integer.



(a) Identify the absolute minimum. [2]

Full solution:
From the graph, the smallest value of $f(x)$ over [2, 10] is when $f(x) = 2$ and $x = 2$.

$\surd$ **The absolute minimum is** ___2___ **at** $x =$ ___2___

◯ There is no absolute minimum.

(b) Identify the absolute maximum. [2]

Full solution:
From the graph, the largest value of $f(x)$ over [2, 10] is when $f(x) = 7$ and $x = 5$.

$\surd$ **The absolute maximum is** ___7___ **at** $x =$ ___5___

◯ There is no absolute maximum.

29. Find the absolute maximum and absolute minimum values of the function $f(x) = x^2 - 8x - 2$ over the interval [0, 7], and indicate the x-values at which they occur.

(a) Identify the absolute maximum. [2]

Full solution:
The absolute maximum and absolute minimum values will occur either at critical points of f or at endpoints of the closed interval [0, 7]. We have:

$$f(x) = x^2 - 8x - 2 \qquad \Rightarrow f'(x) = 2x - 8 = 2(x - 4)$$

The only critical point is at $x = 4$. Check the value the function at this point and the endpoints:

x	0	4	7
f(x)	-2	-18	-9

From the table, the maximum value of f is when $x = 0$.

The absolute maximum value is **-2** at $x =$ **0**

(b) Identify the absolute minimum. [2]

Full solution:

From the table, the minimum value of f is when $x = 4$.

The absolute minimum value is **-18** at $x =$ **4**

30. A fence is to be built to enclose a rectangular area of 1800 square feet. The fence along three sides is to be made of material that costs $4 per foot. The material for the fourth side costs $12 per foot. Find the dimensions of the rectangle that will allow for the most economical fence to be built. [4]

Full solution:

1. Let the side using material costing $12 per foot be x, and the other side be y. Then:

$$\text{Area } = xy = 1800 \quad \Rightarrow y = \frac{1800}{x} \quad x > 0, \quad y > 0$$

and we want to minimize the total cost:

$$\text{Cost } = 12x + 4x + 4y + 4y = 16x + 8y = 16x + 8\frac{1800}{x} = 16x + \frac{14400}{x} = C(x)$$

2. Find the critical points of $C(x)$:

$$C'(x) = 16 - \frac{14400}{x^2} \quad \text{so} \quad C'(x) = 0 \Rightarrow x^2 = \frac{14400}{16} = 900 \Rightarrow x = 30$$

3. By the second-derivative test:

$$C''(x) = \frac{d}{dx}\left(16 - 14400x^{-2}\right) = -14400(-2)x^{-3} = \frac{28800}{x^3}$$

Since $C''(x) > 0$ for all $x > 0$, the function $C(x)$ has an absolute minimum when $x = 30$ and $y = 1800/30 = 60$.

4. The shortest side is therefore 30 ft and the longest side is 60 ft.

The short side is **30** ft and the long side is **60** ft.

31. Find the dimensions of a rectangle with an area of 225 square feet that has the minimum perimeter. [2]

Full solution:

1. Let x and y be the sides of the rectangle. Then

$$\text{Area } = xy = 225 \Rightarrow y = \frac{225}{x} \quad x > 0, \quad y > 0$$

   and we want to minimize the perimeter:

$$\text{Perimeter } = 2x + 2y = 2(x+y) = 2\left(x + \frac{225}{x}\right) = P(x)$$

2. Find the critical points of $P(x)$:

$$P'(x) = 2\left(1 - \frac{225}{x^2}\right) \text{ so } \quad P'(x) = 0 \Rightarrow x^2 = 225 \Rightarrow x = 15$$

3. By the second-derivative test:

$$P''(x) = \frac{d}{dx} 2\left(1 - \frac{225}{x^2}\right) = \frac{900}{x^3}$$

   Since $P''(x) > 0$ for all $x > 0$, the function $P(x)$ has an absolute minimum when $x = 15$ and $y = 225/x = 225/15 = 15$.

4. The dimensions of the rectangle with minimum perimeter are 15 $\times$ 15.

The dimensions of this rectangle are **15** by **15** ft.