MTH 131: Mathematical Analysis for Management, Fall 2017

Practice Midterm 2

Name: ______________________________

Student Number: ______________________________

Answer the questions in the spaces provided on the question sheets.

Show all of your work.

If you run out of room for an answer, continue on the back of the page.

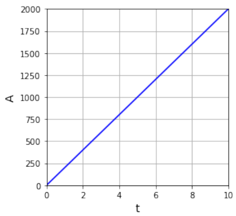
You are allowed to use a non-graphing calculator.

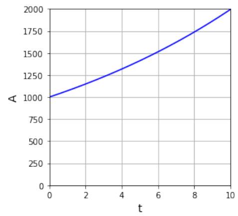
You are allowed a one page formula sheet.

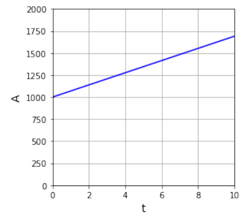
Page	Points	Score
3	10	
4	10	
5	11	
6	12	
7	9	
8	10	
9	12	
10	12	
11	8	
12	6	
Total:	100	

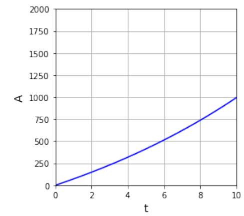
1. If \$1000 is invested at 6.9% compounded continuously, graph the amount in the account as a function of time for a period of 9 years. [1]

   Choose the correct graph.

   ○ 

   ○ 

   ○ 

   ○ 

2. Recently, a certain bank offered a 10-year CD that earns 10.19% compounded continuously.

   (a) If \$30,000 is invested in this CD, how much will it be worth in 10 years? [3]

   Approximately \$ ______________________ (Round to the nearest cent)

   (b) How long will it take for the account to be worth \$75,000? [3]

   Approximately ______________________ years (Round to two decimal places as needed).

3. Find $f'(x)$ for $f(x) = 7e^x + 3x - \ln x$. [3]

   $f'(x) =$ ______________________

4. Find $\dfrac{dy}{dx}$ for $y = 2\log_9 x$. [2]

$\dfrac{dy}{dx} =$ ______________________________

5. Find $\dfrac{dy}{dx}$ for the function $y = 2^x$. [2]

$\dfrac{dy}{dx} =$ ______________________________

6. Find $f'(x)$ for $f(x) = \dfrac{x}{x+18}$. [2]

$f'(x) =$ ______________________________

7. Find $f'(x)$ for $f(x) = 18xe^x$. [2]

$f'(x) =$ ______________________________

8. Find $f'(x)$ for $f(x) = 5x^4 \ln x$. [2]

$f'(x) =$ ______________________________

9. Use the quotient rule to find the derivative of $y = \dfrac{9x^2+7}{x^2+1}$ [3]

$f'(x) =$ ____________________

10. Find $f'(x)$ for $f(x) = (3-5x)^{15}$. [2]

$f'(x) =$ ____________________

11. Find $f'(x)$ for $f(x) = 9\ln(5x^2+6)$. [2]

$f'(x) =$ ____________________

12. For $f(x) = 4e^{x^2-5x+7}$:

    (a) Find $f'(x)$. [2]

    $f'(x) =$ ____________________

    (b) Find the equation of the tangent line to the graph of f when $x = 0$ [2]

    $f'(x) =$ ____________________

(c) Find the value(s) of x where the tangent line is horizontal. [2]

◯ The tangent line is horizontal at x = ____________________

◯ The tangent line is never horizontal.

13. If it is possible to solve for y in terms of x, do so: $3x - 4y = e^y$. [2]

    Select the correct choice below.

◯ y = ____________________

◯ It is impossible to solve the equation for y in terms of x.

14. For the equation $4x^3 - y^2 - 7 = 0$:

    (a) Use implicit differentiation to find y'. [3]

    $y' =$ ____________________

    (b) Evaluate y' at (2, 5). [1]

    $y'(2, 5) =$ ____________________

15. For the equation $y^2 + 3y + 4x = 0$:

    (a) Use implicit differentiation to find y'. [3]

    $y' =$ ____________________

    (b) Evaluate y' at (-7, 4). [1]

    $y'(2, 5) =$ ____________________ (Simplify your answer)

16. Assume that $x = x(t)$ and $y = y(t)$. Let $y = x^3 + 4$ and $\dfrac{dx}{dt} = 4$ when $x = 3$. [3]

Find $\dfrac{dy}{dt}$ when $x = 3$.

$\dfrac{dy}{dt} =$ ____________________ (Simplify your answer)

17. Assume that $x = x(t)$ and $y = y(t)$. Find $\dfrac{dx}{dt}$ using the following information: [3]

$x^2 + y^2 = 585$; $\dfrac{dy}{dt} = -4$ when $x = -21$ and $y = 12$.

$\dfrac{dx}{dt} =$ ____________________ (Type an integer or a simplified fraction)

18. A point is moving on the graph of $xy = 12$. When the point is at (4, 3), its x-coordinate is increasing by 4 units per second. How fast is the y-coordinate changing at that moment? [3]

The y-coordinate is ○ increasing ○ decreasing at __________ units per second.

19. Find the relative rate of change of $f(x) = 15 + 3e^{-3x}$. [2]

The relative rate of change is ________________________

20. For $f(x) = 126 + 32x$:

(a) Find the percentage rate of change of $f(x)$. [2]

The percentage rate of change is ________________________

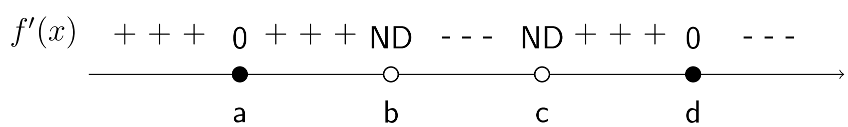
(b) Evaluate the percentage rate of change of $f(x)$ when $x = 6$. [1]

The percentage rate of change when $x = 6$ is ____________ %.

21. Use the price-demand equation $x = f(p) = 30,000 - 700p$ to find $E(p)$, the elasticity of demand. [3]

$E(p) =$ ________________________

22. $f(x)$ is continuous on $(-\infty, \infty)$ and has critical numbers at $x = a$, b, c, and d. Use the sign chart below for $f'(x)$ to determine whether f has a local maximum, a local minimum, or neither at each critical number.



(a) Does $f(x)$ have a local minimum, a local maximum, or no local extremum at $x = a$? [1]

○ a local minimum ○ a local maximum ○ no local extremum

(b) Does $f(x)$ have a local minimum, a local maximum, or no local extremum at $x = b$? [1]

○ a local minimum ○ a local maximum ○ no local extremum

(c) Does $f(x)$ have a local minimum, a local maximum, or no local extremum at $x = c$? [1]

◯ a local minimum ◯ a local maximum ◯ no local extremum

(d) Does $f(x)$ have a local minimum, a local maximum, or no local extremum at $x = d$? [1]

◯ a local minimum ◯ a local maximum ◯ no local extremum

23. Find the intervals on which $f(x)$ is increasing, the intervals on which $f(x)$ is decreasing, and the local extrema for $f(x) = -5x^2 - 30x - 18$.

Type your answers using interval notation, and use a comma to separate answers as needed.

(a) Where is $f(x)$ increasing? [3]

◯ The function is increasing on ________________________

◯ There is no solution.

(b) Where is $f(x)$ decreasing? [3]

◯ The function is decreasing on ________________________

◯ There is no solution.

(c) Which statement is true regarding the local extrema? [2]

◯ The function has a local minimum at $x =$ ________________________

◯ The function has a local maximum at $x =$ ________________________

◯ The function has no local extrema.

24. Find $f''(x)$ for $f(x) = 2x^4 - 3x^3 + 3x - 9$. [2]

$f''(x) =$ ________________________

25. For the function $f(x) = x^{18} + 3x^2$, find the intervals on which the graph of f is concave upward, the intervals on which the graph of f is concave downward, and the inflection points.

(a) For what interval(s) of x is the graph of f concave upward? [2]

◯ The graph is concave upward on the interval(s) ____________________

◯ The graph is never concave upward.

(b) For what interval(s) of x is the graph of f concave downward? [2]

◯ The graph is concave downward on the interval(s) ____________________

◯ The graph is never concave downward.

(c) Determine the x-coordinates of any inflection points of the graph of $f(x)$. Use a comma to separate your answers. [2]

◯ There are inflection points at $x =$ ____________________

◯ There are no inflection points.

26. Use L'Hôpital's rule to find the limit $\lim\limits_{x \to 0} \dfrac{e^{2x} - 1}{3x}$. Use $-\infty$ and ∞ when appropriate. [3]

◯ $\lim\limits_{x \to 0} \dfrac{e^{2x} - 1}{3x} =$ ____________________

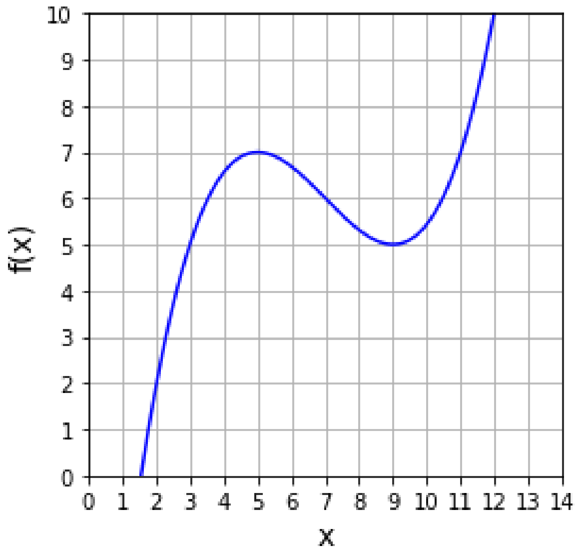
◯ The limit does not exist.

27. Use L'Hôpital's rule to find the limit $\lim\limits_{x \to 0} \dfrac{\ln(1 + 2x)}{x}$. Use $-\infty$ and ∞ when appropriate. [3]

◯ $\lim\limits_{x \to 0} \dfrac{\ln(1 + 2x)}{x} =$ ____________________

◯ The limit does not exist.

28. Refer to the graph shown below. Find the absolute minimum and the absolute maximum over the interval [2, 10]. Round to the nearest integer.



(a) Identify the absolute minimum. [2]

◯ The absolute minimum is ____________ at $x =$ ____________

◯ There is no absolute minimum.

(b) Identify the absolute maximum. [2]

◯ The absolute maximum is ____________ at $x =$ ____________

◯ There is no absolute maximum.

29. Find the absolute maximum and absolute minimum values of the function $f(x) = x^2 - 8x - 2$ over the interval [0, 7], and indicate the x-values at which they occur.

(a) Identify the absolute maximum. [2]

The absolute maximum value is ____________ at $x =$ ____________

(b) Identify the absolute minimum. [2]

The absolute minimum value is ____________ at $x =$ ____________

30. A fence is to be built to enclose a rectangular area of 1800 square feet. The fence along three sides is to be made of material that costs \$4 per foot. The material for the fourth side costs \$12 per foot. Find the dimensions of the rectangle that will allow for the most economical fence to be built. [4]

The short side is ____________ ft and the long side is ____________ ft.

31. Find the dimensions of a rectangle with an area of 225 square feet that has the minimum perimeter. [2]

The dimensions of this rectangle are ____________ by ____________ ft.