MTH 131: Mathematical Analysis for Management, Fall 2017

Practice Midterm 1

Name: _____

Student Number: _____

Answer the questions in the spaces provided on the question sheets.

Show all of your working.

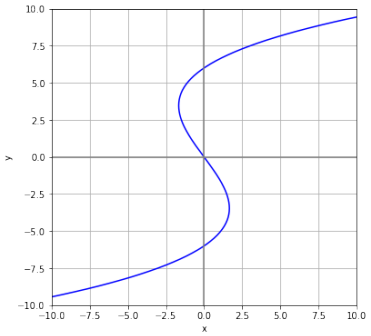
If you run out of room for an answer, continue on the back of the page.

You are allowed to use a non-graphing calculator.

You are allowed a half-page formula sheet.

Page	Points	Score
3	5	
4	9	
5	9	
6	12	
7	7	
8	7	
9	7	
10	4	
11	5	
12	5	
13	9	
14	11	
15	10	
Total:	100	

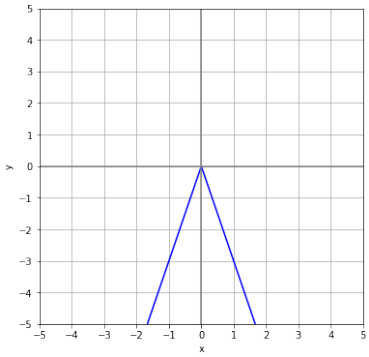
1. Does the following graph specify a function? [1]



◯ The graph does not specify a function.

◯ The graph does specify a function.

2. The graph below involves a reflection in the x-axis and/or a vertical stretch or shrink of a basic function.



(a) Identify the basic function. [1]

◯ $\sqrt{x}$ ◯ x^3 ◯ $|x|$ ◯ x ◯ x^2 ◯ $\sqrt[3]{x}$

(b) Describe the transformation. [1]

◯ The graph is vertically stretched by 3.

◯ The graph is reflected about the x-axis and vertically stretched by 3.

◯ The graph is vertically stretched by $\frac{1}{3}$.

◯ The graph is reflected about the x-axis and vertically stretched by $\frac{1}{3}$.

(c) Write an equation for the graph. [2]

$y =$ _____

3. Find the domain of the function $F(x) = 9x^4 + 6x^2$. Write your answer in interval notation. [2]

The domain of the function is _____

4. Find and simplify each of the following for $f(x) = 5x - 3$.

(a) $f(x+h)$ [2]

$f(x+h) =$ _____

(b) $f(x+h) - f(x)$ [2]

$f(x+h) - f(x) =$ _____

(c) $\dfrac{f(x+h) - f(x)}{h}$ [1]

$\dfrac{f(x+h) - f(x)}{h} =$ _____

5. Use the revenue and cost functions below to answer the following questions.

$$R(x) = 80x - 3x^2 \qquad 1 \leq x \leq 20$$
$$C(x) = 130 + 15x \qquad 1 \leq x \leq 20$$

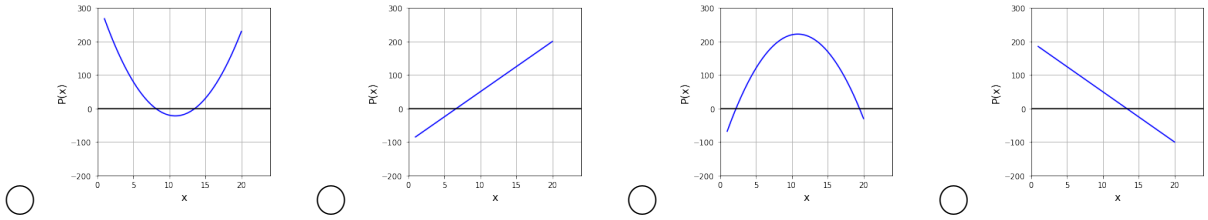
(a) What is the profit function $P(x)$? [2]

$P(x) =$ _____

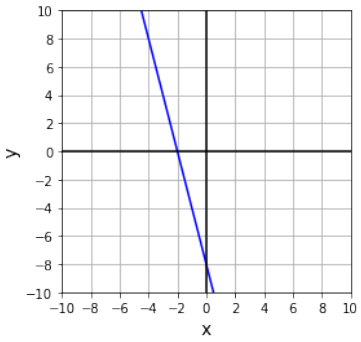
(b) What is the domain of the profit function? [1]

The domain is _____

(c) Choose a possible graph for the profit function. [1]



6. Use the following graph of a line to fill in the answers below.



(a) The x-intercept is _____ [1]

(b) The y-intercept is _____ [1]

(c) The slope is _____ [1]

(d) The slope-intercept form of the equation of the line is _____ [2]

(e) The standard form of the equation of the line is _____ [2]

7. Consider the polynomial function $g(x) = x^2 + 6x + 5$.

(a) The degree of the polynomial is _____ [1]

(b) The y-intercept is _____ [1]

(c) The x-intercept(s) is/are _____ [2]

(d) The equation in vertex form is _____ [4]

(e) The vertex is _____ [1]

(f) The function has a ◯ maximum ◯ minimum [1]

(g) The maximum or minimum value is _____ [1]

(h) The range of the function is _____ [1]

8. A company is planning to manufacture snowboards. The fixed costs are \$100 per day and total costs are \$5900 per day at a daily output of 20 boards.

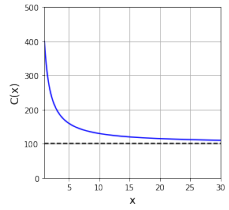
   (a) Assuming that the total cost per day, $C(x)$, is linearly related to the total output per day, x, write an equation for the cost function. [2]

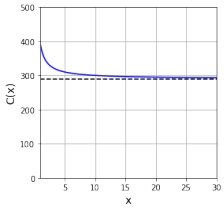
   $C(x) =$ _____

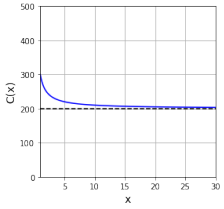
   (b) The average cost per board for an output of x boards is given by $\overline{C}(x) = C(x)/x$. Find the average cost function. [1]

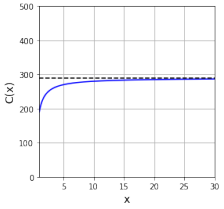
   $\overline{C}(x) =$ _____

   (c) One of the graphs below shows the average cost function, including asymptotes, for $1 \leq x \leq 30$. Choose the correct graph. [1]

   ○ 

   ○ 

   ○ 

   ○ 

   (d) What does the average cost per board approach as production increases? [1]

   The average cost per board approaches \$ _____

9. Solve the given equation for x: [2]

$$g^{4-8x} = g^{3x-5}$$

$x =$ _____ (Write your answer as a fraction or an integer.)

10. Write the expression $\log_6 1296 = 4$ in equivalent exponential form. [1]

The equivalent exponential form is _____

11. Write the equation in equivalent logarithmic form: [1]

$$4 = 16^{\frac{1}{2}}$$

The equivalent logarithmic form is _____

12. What are the domain and range of the function defined by $y = 1 + \ln(x - 7)$? Write your answers in interval notation.

(a) The domain is _____ [2]

(b) The range is _____ [2]

13. Write in terms of simpler forms: $\log_b M^9$ [1]

- ◯ $M + \log_b 9$
- ◯ $9 \log_b M$
- ◯ $9 + \log_b M$
- ◯ $M \log_b 9$

14. How many years will it take \$6,000 to grow to \$9500 if it is invested at 3.75% compounded continuously? [4]

It will take _____ years. (Round to two decimal places.)

15. Find the indicated quantity if it exists.

$$G(x) = \begin{cases} x^2 & \text{for } x < -1 \\ 3x & \text{for } x > -1 \end{cases}$$

(a) Select the correct choice below and fill in any answer boxes in your choice. [1]

- ◯ $\lim_{x \to -1^+} G(x) =$ _____
- ◯ The limit does not exist.

(b) Select the correct choice below and fill in any answer boxes in your choice. [1]

- ◯ $\lim_{x \to -1^-} G(x) =$ _____
- ◯ The limit does not exist.

(c) Select the correct choice below and fill in any answer boxes in your choice. [1]

- ◯ $\lim_{x \to -1} G(x) =$ _____
- ◯ The limit does not exist.

16. If the statement below is always true, explain why. If not, give a counterexample. [1]

"If f is a function such that $\lim_{x\to 0} f(x)$ exists, then $f(0)$ exists."

◯ The statement is not always true. For example, if $f(x) = \dfrac{x}{x^2-1}$, then $\lim_{x\to 0} f(x) = 0$ but $f(0)$ does not exist.

◯ The statement is not always true. For example, if $f(x) = \dfrac{x^2}{x}$, then $\lim_{x\to 0} f(x) = 0$ but $f(0)$ does not exist.

◯ The statement is always true. It is always the case that $\lim_{x\to c} f(x) = f(c)$.

◯ The statement is always true. Although it is possible for $f(0)$ to exist without $\lim_{x\to 0} f(x)$ existing, it is not possible for $\lim_{x\to 0} f(x)$ to exist without $f(0)$ also existing.

17. Consider the limit expression:

$$\lim_{x\to 9} \frac{x^2-4x-45}{x-9}$$

(a) Is the limit expression a $\dfrac{0}{0}$ indeterminate form? Choose the correct answer below. [1]

◯ Yes

◯ No

(b) Select the correct choice below and, if necessary, fill in the answer box with your choice. [2]

◯ $\lim_{x\to 9} \dfrac{x^2-4x-45}{x-9} =$ _____

◯ The limit does not exist and is neither ∞ nor $-\infty$.

18. Find the horizontal and vertical asymptotes for the function $f(x) = \dfrac{x^2+1}{x^2-1}$.

(a) Find the horizontal asymptote(s). [2]

◯ $y =$ _____(Use a comma to separate answers as needed)

◯ There are no horizontal asymptotes.

(b) Find the vertical asymptote(s). [2]

◯ $x =$ _____(Use a comma to separate answers as needed)

◯ There are no horizontal asymptotes.

19. If the statement below is always true, explain why. If not, give a counterexample. [1]

"A polynomial function is continuous for all real numbers"

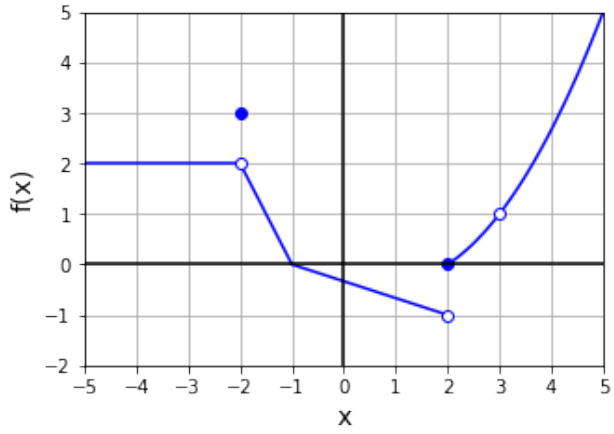
◯ The statement is false. A counterexample is $f(x) = 3x^2 - 2x + 1$.

◯ The statement is false. A counterexample is $f(x) = \dfrac{x^2-4}{x-2}$.

◯ The statement is false. A counterexample is $f(x) = \sqrt{x}$.

◯ The statement is true because, for any positive integer n, x^n is continuous for all real numbers.

20. Use the graph of the function f shown to estimate the indicated quantities to the nearest integer. Select the correct choice in each case and, if necessary, fill in the answer box with your choice.



(a) Find the limit $\lim_{x \to 2^-} f(x)$. [1]

- ◯ $\lim_{x \to 2^-} f(x) =$ _____
- ◯ The limit does not exist.

(b) Find the limit $\lim_{x \to 2^+} f(x)$. [1]

- ◯ $\lim_{x \to 2^+} f(x) =$ _____
- ◯ The limit does not exist.

(c) Find the limit $\lim_{x \to 2} f(x)$. [1]

- ◯ $\lim_{x \to 2} f(x) =$ _____
- ◯ The limit does not exist.

(d) Find the function value $f(2)$. [1]

- ◯ $f(2) =$ _____
- ◯ The value does not exist.

(e) Is f continuous at $x = 2$? [1]

- ◯ Yes
- ◯ No

21. Use the four-step process to find $r'(x)$ for $r(x) = 6 - 2x$. [4]

$r'(x) =$ _____

22. Determine whether f is differentiable at $x = 0$ by considering $\lim_{h \to 0} \dfrac{f(0+h) - f(0)}{h}$: [4]

$$f(x) = 15 - |x|$$

Show all of your work, then choose the correct answer below.

◯ The function f is not differentiable at $x = 0$ because the left- and right-hand limits of the difference quotient exist at $x = 0$, but are not equal.

◯ The function f is differentiable at $x = 0$ because the graph has a sharp corner at $x = 0$.

◯ The function f is differentiable at $x = 0$ because both the left- and right-hand limits of the difference quotient exist at $x = 0$.

◯ The function f is not differentiable at $x = 0$ because the left- and right-hand limits of the difference quotient do not exist at $x = 0$.

23. Find $\dfrac{d}{dx}x^4$ [1]

$\dfrac{d}{dx}x^4 =$ _____

24. Find $\frac{dy}{dx}$ for $y = \frac{1}{x^9}$. [2]

$\frac{dy}{dx} =$ _____

25. Find $f'(t)$ if $f(t) = -7t^2 - 4t + 5$. [3]

$f'(t) =$ _____

26. Find $G'(w)$ if $G(w) = \frac{7}{8w^4} + 9\sqrt{w}$. [4]

$G'(w) =$ _____

27. For $y = f(x) = 3x^5$, find the increments Δx and Δy, and find $\frac{\Delta y}{\Delta x}$, given $x_1 = 1$ and $x_2 = 2$.

(a) Find Δx [1]

$\Delta x =$ _____

(b) Find Δy [1]

$\Delta y =$ _____

(c) Find $\dfrac{\Delta y}{\Delta x}$ [1]

$\dfrac{\Delta y}{\Delta x} =$ _____

28. Find the differential dy [3]

$$y = 28 + 14x^4 - 2x^5$$

$dy =$ _____

29. Find the marginal revenue function. [3]

$$R(x) = x(22 - 0.08x)$$

$R'(x) =$ _____

30. Find the marginal profit function if cost and revenue are given by: [3]

$$C(x) = 231 + 0.8x \qquad \text{and} \qquad R(x) = 8x - 0.09x^2$$

$P'(x) =$ _____