Answer the questions in the spaces provided on the question sheets.

**Show all of your work.**

If you run out of room for an answer, continue on the back of the page.

You are allowed to use a non-graphing calculator.

You are allowed a one page formula sheet.
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1. If $3000 is invested at 8.3% compounded continuously, graph the amount in the account as a function of time for a period of 10 years.

Choose the correct graph.

**Full solution:**
Choose the graph with a y-intercept of $3000 that shows exponential growth.

![Graphs]

2. Recently, a certain bank offered a 5-year CD that earns 6.51% compounded continuously.

(a) If $10,000 is invested in this CD, how much will it be worth in 5 years?

**Full solution:**

\[
A = Pe^{rt} \quad P = 10,000, r = 0.0651, t = 5
\]

\[
= 10000e^{0.0651(5)}
\]

\[
= 83112.69
\]

Approximately $ \boxed{13847.23} \text{ (Round to the nearest cent)}

(b) How long will it take for the account to be worth $25,000?

**Full solution:**

\[
A = Pe^{rt} \quad A = 25,000, P = 10,000, r = 0.0651
\]

\[
25000 = 10000e^{0.0651t}
\]

\[
25000 \quad 10000 = e^{0.0651t}
\]

\[
2.5 = e^{0.0651t}
\]

\[
\ln(2.5) = 0.0651t
\]

\[
t = \ln(2.5)/0.0651
\]

\[
= 14.08
\]

Approximately \boxed{14.08} years (Round to two decimal places as needed).
3. Find $f'(x)$ for $f(x) = 7e^x - \frac{1}{x^8} + 5 \ln x$.

**Full solution:**

$$f'(x) = \frac{d}{dx}(7e^x - \frac{1}{x^8} + 5 \ln x) = 7 \frac{d}{dx} e^x - \frac{d}{dx} x^{-8} + 5 \frac{d}{dx} \ln x = 7e^x + \frac{8}{x^9} + \frac{5}{x}$$

$$f'(x) = 7e^x + \frac{8}{x^9} + \frac{5}{x}$$

4. Find $\frac{dy}{dx}$ for $y = 4 \log_3 x$.

**Full solution:**

$$\frac{d}{dx} \log_b x = \frac{1}{x \ln b}, \quad \text{so} \quad \frac{d}{dx} 4 \log_3 x = 4 \frac{1}{x \ln 3} = \frac{4}{x \ln 3}$$

$$\frac{dy}{dx} = \frac{4}{x \ln 3}$$

5. Find $\frac{dy}{dx}$ for $y = 23^x$.

**Full solution:**

$$\frac{d}{dx} b^x = b^x \ln b, \quad \text{so} \quad \frac{d}{dx} 23^x = 23^x \ln 23$$

$$\frac{dy}{dx} = 23^x \ln 23$$

6. Find $f'(x)$ for $f(x) = 12x^2 e^x$.

**Full solution:**

By product rule,

$$\frac{d}{dx} (12x^2)(e^x) = 12x^2 \frac{d}{dx} (e^x) + e^x \frac{d}{dx} (12x^2) = 12x^2(e^x) + e^x(24x) = 12xe^x(x + 2)$$

$$f'(x) = 12x^2e^x + 24xe^x$$
7. Use the quotient rule to find the derivative of \( y = \frac{6x^2 + 5}{x^2 + 4} \).

**Full solution:**
By quotient rule,
\[
\frac{dy}{dx} = \frac{(x^2 + 4)\frac{d}{dx}(6x^2 + 5) - (6x^2 + 5)\frac{d}{dx}(x^2 + 4)}{(x^2 + 4)^2}
\]
\[
= \frac{(x^2 + 4)(12x) - (6x^2 + 5)(2x)}{(x^2 + 4)^2}
\]
\[
= \frac{12x^3 + 48x - 12x^3 - 10x}{(x^2 + 4)^2}
\]
\[
= \frac{38x}{(x^2 + 4)^2}
\]
\[
y' = \frac{38x}{(x^2 + 4)^2}
\]

8. Find \( f'(x) \) for \( f(x) = (5 - 8\sqrt{x})^{10} \).

**Full solution:**
By general power rule, \( \frac{d}{dx}[u(x)]^n = n[u(x)]^{n-1}u'(x) \). Let \( u(x) = 5 - 8\sqrt{x} \), so \( u'(x) = \frac{d}{dx}(5 - 8x^{1/2}) = -4x^{-1/2} \), and:
\[
\frac{d}{dx}(5 - 8\sqrt{x})^{10} = 10(5 - 8\sqrt{x})^9(-4x^{-1/2}) = -40(5 - 8\sqrt{x})^9/\sqrt{x}
\]
\[
f'(x) = \frac{-40(5 - 8\sqrt{x})^9}{\sqrt{x}}
\]

9. Find \( f'(x) \) for \( f(x) = 7\ln(5 + 6x^2) \).

**Full solution:**
By chain rule, \( \frac{d}{dx}\ln[f(x)] = \frac{1}{f(x)}f'(x) \). Let \( f(x) = 5 + 6x^2 \), so:
\[
\frac{d}{dx}7\ln(5 + 6x^2) = 7\frac{1}{5 + 6x^2}\frac{d}{dx}(5 + 6x^2) = \frac{7}{5 + 6x^2}(12x) = \frac{84x}{5 + 6x^2}
\]
\[
f'(x) = \frac{84x}{5 + 6x^2}
\]
For \( f(x) = \frac{1}{6}e^{2x^3-9x^2+12x+1} \):

(a) Find \( f'(x) \).

**Full solution:**
By chain rule, \( \frac{d}{dx} e^{f(x)} = e^{f(x)} f'(x) \). Let \( f(x) = 2x^3 - 9x^2 + 12x + 1 \), so

\[
\frac{d}{dx} 6e^{2x^3-9x^2+12x+1} = \frac{1}{6} e^{2x^3-9x^2+12x+1} \frac{d}{dx} (2x^3 - 9x^2 + 12x + 1)
= \frac{1}{6} e^{2x^3-9x^2+12x+1} (6x^2 - 18x + 12)
= (x^2 - 3x + 2)e^{2x^3-9x^2+12x+1}
= (x - 1)(x - 2)e^{2x^3-9x^2+12x+1}
\]

\( f'(x) = (x^2 - 3x + 2)e^{2x^3-9x^2+12x+1} \)

(b) Find the equation of the tangent line to the graph of \( f \) when \( x = 0 \)

**Full solution:**

\[
f(0) = \frac{1}{6} e^{2(0)^3-9(0)^2+12(0)+1} = e/6
f'(0) = ((0)^2 - 3(0) + 2)e^{2(0)^3-9(0)^2+12(0)+1} = 2e
\]

Plugging into the slope-intercept form \( y = mx + b \) with \( m = 2e, b = e/6 \), yields

\[ y = 2ex + e/6 \]

(c) Find the value(s) of \( x \) where the tangent line is horizontal.

**Full solution:**
The tangent line is horizontal when \( f'(x) = 0 \), so \( (x - 1)(x - 2)e^{2x^3-9x^2+12x+1} = 0 \). Since \( e^{2x^3-9x^2+12x+1} > 0 \) for all \( x \), this means \( (x - 1)(x - 2) = 0 \), so \( x = 1 \) or \( x = 2 \).

√ The tangent line is horizontal at \( x = 1, 2 \)

○ The tangent line is never horizontal.

11. If it is possible to solve for \( y \) in terms of \( x \), do so: \( 4x + 6y = e^y \).

**Full solution:**
Can't be done.

○ \( y = \) __________________________

√ It is impossible to solve the equation for \( y \) in terms of \( x \).
12. For the equation \( y^2 + 4y + 5x = 0 \):

(a) Use implicit differentiation to find \( y' \).

**Full solution:**

Differentiate each term with respect to \( x \), using the chain rule for \( y^2 \):

\[
\frac{dy}{dx}(y^2 + 4y + 5x) = \frac{d}{dy}(y^2)\frac{dy}{dx} + 4\frac{dy}{dx} + 5\frac{d}{dx}x = 2yy' + 4y' + 5 = 0
\]

Solve for \( y' \):

\[2yy' + 4y' + 5 = 0 \Rightarrow (2y + 4)y' = -5 \Rightarrow y' = \frac{-5}{2y + 4}\]

\[y' = \frac{-5}{2y + 4}\]

(b) Evaluate \( y' \) at the point \((-1, 1)\).

**Full solution:**

\[\frac{-5}{2y + 4} = \frac{-5}{2(1) + 4} = \frac{-5}{6}\]

\[y'(-1, 1) = \frac{-5}{6}\]

13. Assume that \( x = x(t) \) and \( y = y(t) \). Find \( \frac{dx}{dt} \) using the following information:

\( x^2 + y^2 = 5.8; \ \frac{dy}{dt} = -2 \) when \( x = -1.8 \) and \( y = 1.6 \).

**Full solution:**

Differentiate each term with respect to \( t \), using the chain rule for \( x^2 \) and \( y^2 \):

\[
\frac{d}{dx}(x^2)\frac{dx}{dt} + \frac{d}{dy}(y^2)\frac{dy}{dt} = 0 \Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0
\]

Find \( \frac{dx}{dt} \) when \( \frac{dy}{dt} = -2 \), \( x = -1.8 \), and \( y = 1.6 \):

\[2(-1.8)\frac{dx}{dt} + 2(1.6)(-2) = 0 \Rightarrow \frac{dx}{dt} = \frac{-3.2/1.8}{-16/9} = \frac{17}{16} = 1.7\]

\[\frac{dx}{dt} = \frac{-16/9 = 1.7}{16/9 = 1.7}\]
14. For \( f(x) = 195 + 54x \):

(a) Find the percentage rate of change of \( f(x) \).

**Full solution:**

The percentage rate of change of \( f(x) \) is

\[
100 \frac{f'(x)}{f(x)} = 100 \frac{54}{195 + 54x} = \frac{5400}{195 + 54x}
\]

The percentage rate of change is \( \frac{5400}{195 + 54x} \).

(b) Evaluate the percentage rate of change of \( f(x) \) when \( x = 5 \).

**Full solution:**

\[
\frac{5400}{195 + 54(5)} = \frac{5400}{465} = 11.6\%
\]

The percentage rate of change when \( x = 5 \) is 11.6\% (Round to 1 decimal place).

15. Using the price-demand equation \( x = f(p) = 15,000 - 650p \):

(a) Find \( E(p) \), the elasticity of demand.

**Full solution:**

\[
E(p) = -\frac{p f'(p)}{f(p)} = -\frac{p(-650)}{15000 - 650p} = \frac{13p}{300 - 13p}
\]

\( E(p) = \frac{13p}{300 - 13p} \)

(b) Evaluate \( E(p) \) when \( p = 20 \).

**Full solution:**

\[
E(20) = \frac{13(20)}{300 - 13(20)} = \frac{260}{40} = 6.5
\]

\( E(20) = 6.5 \)

(c) Is the demand inelastic, elastic, or have unit elasticity when \( p = 20 \)?

**Full solution:**

Demand is inelastic if \( 0 < E(p) < 1 \), elastic if \( E(p) > 1 \), and unit if \( E(p) = 1 \).

- Inelastic
- Elastic
- Unit elasticity
(d) If prices are increased when \( p = 20 \), will revenues increase, decrease, or stay the same?  

**Full solution:**  
When demand is elastic, price increases cause revenues to decrease.  
- Revenues will increase  
- **√ Revenues will decrease**  
- Revenues will stay the same  

16. \( f(x) \) is continuous on \((-\infty, \infty)\) and has critical numbers at \( x = a, b, c, \) and \( d \). Use the sign chart below for \( f'(x) \) to determine whether \( f \) has a local maximum, a local minimum, or neither at each critical number.  

\[
\begin{array}{cccccccc}
\text{f}'(x) & - & - & 0 & + & + & ND & + & + & ND & - & - & 0 & - & - \\
\hline
& a & b & c & d
\end{array}
\]

(a) Does \( f(x) \) have a local minimum, a local maximum, or no local extremum at \( x = a \)\?  

**Full solution:**  
\( f(x) \) changes from decreasing to increasing at \( x = c \), so this is a local minimum.  
- **√ a local minimum**  
- a local maximum  
- no local extremum  

(b) Does \( f(x) \) have a local minimum, a local maximum, or no local extremum at \( x = b \)?  

**Full solution:**  
\( f'(x) \) does not change sign at \( x = a \), so this is not a local extremum.  
- a local minimum  
- a local maximum  
- **√ no local extremum**  

(c) Does \( f(x) \) have a local minimum, a local maximum, or no local extremum at \( x = c \)?  

**Full solution:**  
\( f(x) \) changes from increasing to decreasing at \( x = b \), so this is a local maximum.  
- a local minimum  
- **√ a local maximum**  
- no local extremum  

(d) Does \( f(x) \) have a local minimum, a local maximum, or no local extremum at \( x = d \)?  

**Full solution:**  
\( f'(x) \) does not change sign at \( x = a \), so this is not a local extremum.  
- a local minimum  
- a local maximum  
- **√ no local extremum**
17. Find the intervals on which \( f(x) \) is increasing, the intervals on which \( f(x) \) is decreasing, and the local extrema for \( f(x) = -3x^2 - 24x - 20 \).

Type your answers using interval notation, and use a comma to separate answers as needed.

(a) Where is \( f(x) \) increasing?

**Full solution:**
The function is increasing when \( f'(x) > 0 \). Since \( f'(x) = -6x - 24 = -6(x + 4) \), then:
\[
f'(x) > 0 \Rightarrow -6(x + 4) > 0 \Rightarrow x + 4 < 0 \Rightarrow x < -4
\]
\( \checkmark \) The function is increasing on \( (-\infty, -4) \)

(b) Where is \( f(x) \) decreasing?

**Full solution:**
The function is decreasing when \( f'(x) < 0 \):
\[
f'(x) < 0 \Rightarrow -6(x + 4) < 0 \Rightarrow x + 4 > 0 \Rightarrow x > -4
\]
\( \checkmark \) The function is decreasing on \( (-4, \infty) \)

(c) Which statement is true regarding the local extrema?

**Full solution:**
The function changes increasing \( \rightarrow \) decreasing at \( x = -4 \), so this is a local maximum.

\( \checkmark \) The function has a local maximum at \( x = -4 \)

18. Find the second derivative for the function \( f(x) = 2x^3 + 3 \ln x + e^x \).

**Full solution:**
\[
f(x) = 2x^3 + 3 \ln x + e^x
\]
\[
f'(x) = 6x^2 + \frac{3}{x} + e^x = 6x^2 + 3x^{-1} + e^x
\]
\[
f''(x) = 12x - \frac{3}{x^2} + e^x = 12x - 3x^{-2} + e^x
\]
\( f''(x) = 12x - 3/x^2 + e^x \)
19. For the function $f(x) = x^3 - 3x^2 + 3x - 1$, find the intervals on which the graph of $f$ is concave upward, the intervals on which the graph of $f$ is concave downward, and the inflection points.

(a) For what interval(s) of $x$ is the graph of $f$ concave upward?

Full solution:
Concavity depends on the sign of the second derivative:

$$f(x) = x^3 - 3x^2 + 3x - 1 \Rightarrow f'(x) = 3x^2 - 6x + 3 \Rightarrow f''(x) = 6x - 6 = 6(x - 1)$$

$f''(x) > 0$ when $x > 1$, so the graph of $f$ is concave upward for $x > 1$.

√ The graph is concave upward on the interval(s) $(1, \infty)$

○ The graph is never concave upward.

(b) For what interval(s) of $x$ is the graph of $f$ concave downward?

Full solution:
$f''(x) < 0$ for $x < 1$, so the graph of $f$ is concave downward for $x < 1$.

√ The graph is concave downward on the interval(s) $(-\infty, 1)$

○ The graph is never concave downward.

(c) Determine the x-coordinates of any inflection points of the graph of $f(x)$. Use a comma to separate your answers.

Full solution:
Concavity changes when $x = 1$, so there is an inflection point when $x = 1$.

√ There are inflection points at $x = 1$

○ There are no inflection points.

20. Use L'Hôpital's rule to find the limit $\lim_{x \to 9} \frac{x^2 - x - 72}{x - 9}$. Use $-\infty$ and $\infty$ when appropriate.

Full solution:

$$\lim_{x \to 9} x^2 - x - 72 = 9^2 - 9 - 72 = 0 \quad \text{and} \quad \lim_{x \to 9} x - 9 = 9 - 9 = 0$$

So, L'Hôpital's rule applies:

$$\lim \lim_{x \to 9} \frac{x^2 - x - 72}{x - 9} = \frac{d}{dx}(x^2 - x - 72) \quad \text{L'Hôp} \quad \lim_{x \to 9} \frac{d}{dx}(x^2 - x - 72) = \lim_{x \to 9} \frac{2x - 1}{1} = \frac{2(9) - 1}{1} = 17$$

√ $\lim_{x \to 9} \frac{x^2 - x - 72}{x - 9} = 17$

○ The limit does not exist.
21. Use L'Hôpital's rule to find the limit \( \lim_{x \to 0} \frac{e^{3x} - 1}{2x} \). Use \(-\infty\) and \(\infty\) when appropriate.

**Full solution:**

\[
\lim_{x \to 0} e^{3x} - 1 = e^{3(0)} - 1 = 0 \quad \text{and} \quad \lim_{x \to 0} 2x = 2(0) = 0
\]

So, L'Hôpital's rule applies:

\[
\lim_{x \to 0} \frac{e^{3x} - 1}{2x} \overset{\text{L'Hôp}}{=} \lim_{x \to 0} \frac{\frac{d}{dx}(e^{3x} - 1)}{\frac{d}{dx}(2x)} = \lim_{x \to 0} \frac{3e^{3x}}{2} = \frac{3e^0}{2} = \frac{3}{2}
\]

\[\sqrt{\lim_{x \to 0} \frac{e^{3x} - 1}{2x}} = \frac{3}{2}\]

○ The limit does not exist.

22. Refer to the graph shown below. Find the absolute minimum and the absolute maximum over the interval \([7, 12]\). Round to the nearest integer.

(a) Identify the absolute minimum.

**Full solution:**

From the graph, the smallest value of \( f(x) \) over \([7, 12]\) is when \( f(x) = 0 \) and \( x = 12 \).

\[\sqrt{\text{The absolute minimum is } 0 \text{ at } x = 12}\]

○ There is no absolute minimum.
(b) Identify the absolute maximum.

**Full solution:**
From the graph, the largest value of $f(x)$ over $[7, 12]$ is when $f(x) = 5$ and $x = 9$.

✓ The absolute maximum is $5$ at $x = 9$
⊙ There is no absolute maximum.

23. Find the absolute maximum and absolute minimum values of the function $f(x) = x^2 - 10x - 8$ over the interval $[1, 8]$, and indicate the x-values at which they occur.

(a) Identify the absolute maximum.

**Full solution:**
The absolute maximum and absolute minimum values will occur either at critical points of $f$ or at endpoints of the closed interval $[0, 7]$. We have:

$$f(x) = x^2 - 10x - 8 \Rightarrow f'(x) = 2x - 10 = 2(x - 5)$$

The only critical point is at $x = 5$. Check the value the function at this point and the endpoints:

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<tr>
<td>$f(x)$</td>
<td>-17</td>
<td>-33</td>
<td>-24</td>
</tr>
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From the table, the maximum value of $f$ is when $x = 1$.

The absolute maximum value is $-17$ at $x = 1$

(b) Identify the absolute minimum.

**Full solution:**
From the table, the minimum value of $f$ is when $x = 5$.

The absolute minimum value is $-33$ at $x = 5$
24. A wall is to be built to enclose a rectangular area of 300 square feet. The wall along three sides is to be built of brick that costs $4 per foot. The fourth wall is to be built of wood that costs $2 per foot. Find the dimensions of the rectangle that will allow for the least expensive wall to be built, and the total cost of the wall.

**Full solution:**

1. Let the side made of wood costing $2 per foot be \( x \), and the other side be \( y \). Then:

\[
\text{Area} = xy = 300 \quad \Rightarrow \quad y = \frac{300}{x} \quad x > 0, \quad y > 0
\]

and we want to minimize the total cost:

\[
\text{Cost} = 2x + 4x + 4y + 4y = 6x + 8y = 6x + 8\frac{300}{x} = 6x + \frac{2400}{x} = C(x)
\]

2. Find the critical points of \( C(x) \):

\[
C'(x) = 6 - \frac{2400}{x^2} \quad \text{so} \quad C'(x) = 0 \Rightarrow x^2 = \frac{2400}{6} = 400 \Rightarrow x = 20
\]

3. By the second-derivative test:

\[
C''(x) = \frac{d}{dx} \left( 6 - 2400x^{-2} \right) = -2400(-2)x^{-3} = \frac{24800}{x^3}
\]

Since \( C''(x) > 0 \) for all \( x > 0 \), the function \( C(x) \) has an absolute minimum when \( x = 20 \) and \( y = \frac{300}{20} = 15 \).

4. The wood side is therefore 20 ft and the other side is 15 ft.

5. The total cost of the wall is \( C(20) = 6(20) + \frac{2400}{20} = 120 + 120 = $240 \).

(a) The wall made of wood is \( \boxed{20} \) ft long and the other side is \( \boxed{15} \) ft long. \[6\]

(b) The total cost of the wall is $ \( \boxed{240} \). \[2\]