MTH 131: Mathematical Analysis for Management, Fall 2017

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Midterm 1 Answer Key

Name: ______________________________

Student Number: ______________________________

Answer the questions in the spaces provided on the question sheets.

Show all of your work.

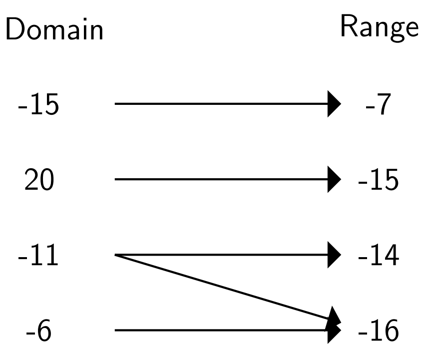
If you run out of room for an answer, continue on the back of the page.

You are allowed to use a non-graphing calculator.

You are allowed a half-page formula sheet.

Page	Points	Score
3	7	
4	12	
5	14	
6	8	
7	11	
8	12	
9	9	
10	15	
11	12	
Total:	100	

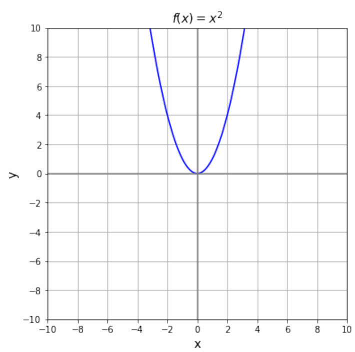
1. Does the following table specify a function? [1]



○ The table does specify a function.

√ **The table does not specify a function.**

2. How is the graph of $g(x) = (x+2)^2 + 3$ related to the graph of $f(x) = x^2$ shown below? [4]



The graph of $g(x) = (x+2)^2 + 3$ is the same as the graph of $f(x) = x^2$ shifted to the √ **left** ○ right ___**2**___ units and shifted √ **up** ○ down ___**3**___ units.

3. Use interval notation to write the solution set of the inequality below. [2]

$$5x + 2 < -5$$

Full solution:

$$5x + 2 < -5 \Rightarrow 5x < -7 \Rightarrow x < -7/5$$

The solution set is ___**$(-\infty, -7/5)$**___ (Use integers or fractions for any numbers.)

4. (a) Find the slope of the line that passes through the points (7, 1) and (14, 5). [2]

Full solution:

$$m = \frac{5-1}{14-7} = \frac{4}{7}$$

√ **m = ________4/7________ (Use an integer or fraction.)**

◯ The slope is not defined.

(b) What is the equation of the line in point-slope form? [2]

Full solution: Point-slope form is: $y - y_1 = m(x - x_1)$

√ **____y - 1 = 4/7(x - 7)____ (Use integers or fractions for any numbers.)**

◯ There is no point-slope form.

(c) What is the equation of the line in slope-intercept form? [2]

Full solution: Slope-intercept form is: $y = mx + b$.

√ **____y = 4/7 x - 3____ (Use integers or fractions for any numbers.)**

◯ There is no slope-intercept form.

(d) What is the equation of the line in standard form? [2]

Full solution: Standard form is $Ax + By = C$.

____**4x - 7y = 21**____ (Use integers or fractions for any numbers.)

5. Solve the given equation for x. Write your answer as a fraction or an integer [4]

$$8^{-5x-x^2} = 8^4$$

Full solution:

$$\begin{aligned} 8^{-5x-x^2} &= 8^4 \\ -5x - x^2 &= 4 \\ x^2 + 5x + 4 &= 0 \\ (x+1)(x+4) &= 0 \end{aligned}$$

$x =$ ________**-1, -4**________ (Use a comma to separate answers.)

6. Find the value of an investment of \$25,000 for 14 years at an annual interest rate of 3.15%. Do not round until the final answer, then round your answers to the nearest cent.

   (a) Compounded monthly. [5]

   Full solution:

   $$\begin{aligned} A &= P(1 + r/m)^{mt} \\ &= 25000(1 + 0.0315/12)^{12\cdot 14} \\ &\approx 38834.07 \end{aligned}$$

   The value of the investment is \$ **38,834.07**

   (b) Compounded continuously. [5]

   Full solution:

   $$\begin{aligned} A &= Pe^{rt} \\ &= 25000e^{0.0315\cdot 14} \\ &\approx 38856.52 \end{aligned}$$

   The value of the investment is \$ **38,856.52**

7. Write the expression $\log_2 2048 = 11$ in equivalent exponential form. [2]

   Full solution: $\log_b x = y$ is equivalent to $x = b^y$.

   The equivalent exponential form is $\mathbf{2048 = 2^{11}}$

8. Write the expression $343 = 7^3$ in equivalent logarithmic form. [2]

   Full solution: $x = b^y$ is equivalent to $\log_b x = y$.

   The equivalent logarithmic form is $\mathbf{\log_7 343 = 3}$

9. If the following statement is true, explain why. If not, give a counterexample. [1]

"Every polynomial function is one-to-one."

Full solution: A horizontal line may cross the graph of a polynomial more than once.

◯ The statement is false. A counterexample is f(x) = x^3, where each range value, with the exception of 0, has 2 corresponding domain values.

◯ The statement is true because every range value of a polynomial corresponds to more than one domain value.

√ **The statement is false. A counterexample is f(x) = x^2, where each range value, with the exception of 0, has 2 corresponding domain values.**

◯ The statement is true because every range value of a polynomial corresponds to exactly one domain value.

10. How many years will it take $15,000 to grow to $24,000 if it is invested at an annual interest rate of 4.5% compounded weekly? Round your answer to two decimal places. [5]

Full solution: $A = P(1 + r/m)^{mt}$ with $A = 24000$, $P = 15000$, $r = 0.045$, $m = 52$:

$$\begin{aligned} 24000 &= 15000(1 + 0.045/52)^{52t} \\ 24000/15000 &= (1 + 0.045/52)^{52t} \\ \ln(24000/15000) &= 52t \ln(1 + 0.045/52) \\ t &= [\ln 24000/15000]/[52 \ln(1 + 0.045/52)] \approx 10.45 \end{aligned}$$

It will take ____**10.45**____ years.

11. Let f(x) = $\frac{\text{x - 2}}{|\text{x - 2}|}$.

(a) Find the limit $\lim_{x \to 2^+} f(x)$. [2]

Full solution: $\lim_{x \to 2^+} \frac{x-2}{|x-2|} = \lim_{x \to 2^+} \frac{x-2}{x-2} = 1$

√ $\lim_{x \to 2^+} f(x) =$ ____**1**____

◯ The limit does not exist.

(b) Find the limit $\lim_{x \to 2^-} f(x)$. [2]

Full solution:

$$\lim_{x \to 2^-} \frac{x-2}{|x-2|} = \lim_{x \to 2^+} \frac{x-2}{-(x-2)} = -1$$

√ $\lim_{x \to 2^-} f(x) =$ **-1**

◯ The limit does not exist.

(c) Find the limit $\lim_{x \to 2} f(x)$. [2]

Full solution: The left-hand and right hand limits are different, so the limit does not exist.

◯ $\lim_{x \to 2} f(x) =$ ____________________

√ **The limit does not exist.**

(d) Find the function value $f(2)$. [2]

Full solution: f(2) does not exist, since f(2) = 0/0.

◯ $f(2) =$ ____________________

√ **f(2) does not exist.**

12. For the function $f(x) = \dfrac{x^2 + x - 6}{x^2 + 2x - 3}$.

(a) Find the zeros of the denominator. Use a comma to separate answers as needed. [3]

Full solution:

$$x^2 + 2x - 3 = (x+3)(x-1)$$

The zeros of the denominator are **-3, 1**

(b) Find the domain of the function. Write your answer using interval notation. [2]

Full solution: The domain of a rational function is the set of all real number such that the denominator is nonzero.

The domain of the function is $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$

(c) Find the x-intercept(s). Use a comma to separate answers as needed. [2]

Full solution: For the numerator,

$$x^2 + x - 6 = (x + 3)(x - 2)$$

However, -3 is not in the domain of the function.

The x-intercept(s) are **2**

(d) Find the y-intercept. [2]

Full solution: The y-intercept is given by f(0) = -6/-3 = 2

The y-intercept is **2**

(e) Find the horizontal asymptote(s). Use a comma to separate answers as needed. [2]

Full solution: The limit of a rational function at $\pm\infty$ depends only on the leading terms of the polynomials in the numerator and denominator:

$$\lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to\infty} \frac{x^2}{x^2} = 1$$

√ $y =$ **1**

◯ There are no horizontal asymptotes.

(f) Find the vertical asymptote(s). Use a comma to separate answers as needed [3]

Full solution: Vertical asymptotes occur in a rational function where the denominator is zero but the numerator is nonzero. The denominator is zero at $x = -3$ and $x = 1$, but $x = -3$ is not a vertical asymptote since $\lim_{x\to-3} = 5/4$, not $\pm\infty$.

√ $x =$ **1**

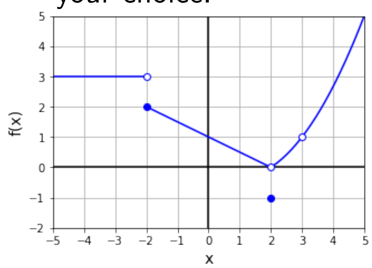
◯ There are no vertical asymptotes.

(g) Where is the function continuous? Write your answer using interval notation. [3]

Full solution: A rational function is continuous for all x except those values that make a denominator zero.

The function is continuous on the interval(s) $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$

13. Use the graph of the function f shown to estimate the indicated quantities to the nearest integer. Select the correct choice in each case and, if necessary, fill in the answer box with your choice.



(a) Find the limit $\lim_{x \to 2^-} f(x)$. [2]

Full solution: Look at the value $f(x)$ approaches as x approaches 2 from the left.

√ $\lim_{x \to 2^-} f(x) =$ **0**

◯ The limit does not exist.

(b) Find the limit $\lim_{x \to 2^+} f(x)$. [2]

Full solution: Look at the value $f(x)$ approaches as x approaches 2 from the right.

√ $\lim_{x \to 2^+} f(x) =$ **0**

◯ The limit does not exist.

(c) Find the limit $\lim_{x \to 2} f(x)$. [2]

Full solution: The left-hand and right-hand limits are the same, so the limit exists.

√ $\lim_{x \to 2} f(x) =$ **0**

◯ The limit does not exist.

(d) Find the function value $f(2)$. [2]

Full solution: $f(2) = 0$, since there is a filled dot on the graph at the point (2, -1).

√ $f(2) =$ **-1**

◯ The value does not exist.

(e) Is f continuous at $x = 2$? [1]

Full solution: f is not continuous at $x = 2$ since $\lim_{x \to 2} f(x) \neq f(2)$.

√ **Yes**

◯ No

14. If the statement is always true, explain why. If not, give a counterexample. [2]

"If f is a function that is continuous at x = 0 and x = 2, then f is continuous at x = 1."

Full solution: Continuity at one point implies nothing about continuity at any other points.

◯ The statement is false. A counterexample is a function with a jump discontinuity at x = 0, for example, y = 5/x.

√ **The statement is false. A counterexample is a function with a point discontinuity at x = 1, for example, y = 1/(x - 1).**

◯ The statement is false. A counterexample is a function with a jump discontinuity at x = 2, for example, y = 3/(x - 2).

◯ The statement is true because, if f is continuous at points a and b, then f is continuous on the interval (a, b).

15. Find $\frac{d}{dx}\left(\frac{5x^2}{7} - \frac{4}{3x^2}\right)$ [5]

Full solution: Write as $y = \frac{5}{7}x^2 - \frac{4}{3}x^{-2}$, then apply the Power Rule.

$\frac{d}{dx}\left(\frac{5x^2}{7} - \frac{4}{3x^2}\right) =$ **(10/7)x + (8/3)x^{-3} = 10x/7 + 8/(3x^3)**

16. For f(x) = $\sqrt{x}$:

(a) Find f'(x). [3]

Full solution: $f(x) = x^{1/2} \Rightarrow f'(x) = \frac{1}{2}x^{-1/2}$

f'(x) = **$1/2\sqrt{x}$**

(b) Find the slope of the graph of f at x = 4. [2]

Full solution: $f'(4) = 1/2\sqrt{4} = 1/4$

The slope is **1/4**

(c) Find the equation of the tangent line to f at x = 4. [3]

Full solution: $y - 2 = \frac{1}{4}(x - 4) \Rightarrow y = \frac{1}{4}x + 1$

The equation of the tangent line is **y = x/4 + 1**

17. The cost and revenue functions from the sale of x lawn mowers are:

$$C(x) = 450 + 10x \qquad \text{and} \qquad R(x) = -0.06x^2 + 70x$$

(a) Find the profit function $P(x)$. [2]

Full solution: $P(x) = R(x) - C(x) = -0.06x^2 + 60x - 450$

$P(x) =$ **$-0.06x^2 + 60x - 450$**

(b) Find the average profit per mower, $\overline{P}(x)$. [2]

Full solution: $\overline{P}(x) = P(x)/x = -0.06x + 60 - 450/x$

$\overline{P}(x) =$ **$-0.06x + 60 - 450/x$**

(c) Find the average profit per mower at a production level of 20 units. [1]

Full solution: $\overline{P}(20) = -0.06(20) + 60 - 450/20 = 36.30$

The average profit per mower when $x = 20$ is $ **36.30**

(d) Find the marginal average profit per mower. [2]

Full solution: $\overline{P}'(x) = -0.06x - 450/x^2$

The marginal average profit per mower is **$-0.06 + 450/x^2$**

(e) Find the marginal average profit per mower at a production level of 20 units. [1]

Full solution: $\overline{P}'(20) = -0.06 + 450/20^2 =$

The marginal average profit per mower when $x = 20$ is $ **1.07**

(f) Using parts (C) and (E), estimate the average profit per mower if 21 mowers are produced. [3]

Full solution: $\overline{P}(21) \approx \overline{P}(20) + \overline{P}'(20)$

The estimated average profit per mower when $x = 21$ is $ **37.37**

(g) Choose the correct interpretation of $\overline{P}'(20) = \$1.07$ per mower. [1]

- √ **At a production level of 20 mowers the average profit is increasing at a rate of $1.07 per mower.**
- ◯ At a production level of 20 mowers a unit increase in production will increase the total profit by $1.07 per mower.
- ◯ At a production level of 20 mowers the average profit is $1.07 per mower.