

Bernoulli

- Two possible outcomes: **success**, denoted by s , and **failure**, denoted by f .
- Sample space contains two points, s and f .
- The random variable $X(s) = 1$ and $X(f) = 0$ is a **Bernoulli random variable**.

Parameters

p = prob. of success
 $q = 1 - p$ = prob. of failure

Probability Mass Function

$$P(X = x) = \begin{cases} 1 - p \equiv q & \text{if } x = 0 \\ p & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

Expectation

p

Variance

$p(1 - p)$

Binomial

- Independent Bernoulli trials repeated n times.
- Sample space is set of different sequences of length n .
- Number of successes is a random variable X , called a **binomial with parameters n and p** .

Parameters

p = prob. of success
 n = number of trials

Probability Mass Function

$$P(X = x) = \begin{cases} \binom{n}{x} p^x (1 - p)^{n-x} & \text{if } x = 0, 1, \dots, n \\ 0 & \text{elsewhere} \end{cases}$$

Expectation

np

Variance

$np(1 - p)$

Discrete Uniform

- Sample space is set $\{1, 2, \dots, N\}$.
- The integer chosen is a random variable X .

Parameters

N = number of choices

Probability Mass Function

$$P(X = x) = \begin{cases} \frac{1}{N} & \text{if } x = 1, \dots, N \\ 0 & \text{elsewhere} \end{cases}$$

Expectation

$\frac{N + 1}{2}$

Variance

$\frac{(N + 1)(N - 1)}{12}$

Geometric

- Independent Bernoulli trials are repeated until the first success occurs.
- Sample space is $S = \{s, fs, ffs, fffs, \dots, ff \cdots fs, \dots\}$.
- Number of experiments until the first success is a discrete random variable X , called **geometric**.

Parameters

p = prob. of success

Probability Mass Function

$$P(X = x) = \begin{cases} (1 - p)^{x-1} p & x = 1, 2, 3, \dots, \\ 0 & \text{elsewhere} \end{cases}$$

Expectation

$\frac{1}{p}$

Variance

$\frac{1 - p}{p^2}$

Hypergeometric

- A box contains D defective and $N - D$ nondefective items.
- Items are drawn at random without replacement, not exceeding D or $N - D$.
- Number of defective items drawn is a discrete random variable X , called **hypergeometric**.

Parameters	Probability Mass Function	Expectation	Variance
$D =$ defective $N =$ total	$P(X = x) = \begin{cases} \frac{\binom{D}{x} \binom{N-D}{n-x}}{\binom{N}{n}} & 0 \leq x \leq n \\ 0 & \text{elsewhere} \end{cases}$	$\frac{nD}{N}$	$\frac{nD(N-D)}{N^2} \left(1 - \frac{n-1}{N-1}\right)$

Negative Binomial

- Sequence of independent Bernoulli trials, each with probability of success p , $0 < p < 1$.
- Number of experiments until r th success is a discrete random variable X , called **negative binomial**.

Parameters	Probability Mass Function	Expectation	Variance
$p =$ prob. of success $r =$ number of trials	$P(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, x = r, r+1, \dots,$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$

Poisson

- Models number of times some event occurs in a given time interval.
- Poisson approximates the Binomial when n is large and p is small.

Parameters	Probability Mass Function	Expectation	Variance
$\lambda = np$	$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, 3, \dots$	λ	λ