Beta

- Random variable that varies between two finite limits.
- Median of $(2n+1)$ numbers from the interval (0, 1) is a beta random variable with parameters $(n+1, n+1)$.

Parameters	Probability Density Function	Expectation	Variance
$\alpha > 0$, $\beta > 0$	$f(x) = \begin{cases} \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

Cauchy

- Let Z_1 and Z_2 be two standard normal distributions.
- The ratio Z_1/Z_2 has a Cauchy distribution.
- Special case of Student's t distribution when degrees of freedom $= 1$.

Parameters	Probability Density Function	Expectation	Variance
$-\infty < \theta < \infty$, $\sigma > 0$	$f(x) = \frac{1}{\pi\sigma} \frac{1}{1 + \left(\frac{x-\theta}{\sigma}\right)^2}$	Does not exist	Does not exist

Chi Squared

- The square of a standard normal random variable has a chi squared distribution with $p = 1$.
- Special case of the Gamma distribution with $\alpha = p/2$, p an integer, and $\beta = 2$.

Parameters	Probability Density Function	Expectation	Variance
$p = 1, 2, \dots$	$f(x) = \begin{cases} \frac{1}{\Gamma(p/2) 2^{p/2}} x^{(p/2)-1} e^{-x/2} & x \ge 0 \\ 0 & x < 0 \end{cases}$	p	$2p$

Double Exponential

- Symmetric distribution centered on the parameter μ.
- Has the form of two exponential distributions joined "back-to-back".

Parameters	Probability Density Function	Expectation	Variance
μ, $\sigma > 0$	$f(x) = \frac{1}{2\sigma} e^{-\lvert x-\mu \rvert/\sigma}$	μ	$2\sigma^2$

Exponential

- $\{N(t) : t \geq 0\}$ is a Poisson process. $N(t)$ is number of events at or prior to time t.
- X_1 is time of first event, X_2 is elapsed time between first and second event etc.
- **Interarrival time** is a random variable X.
- Special case of Gamma distribution with $\alpha = 1$.

Parameters	Probability Density Function	Expectation	Variance
$\lambda > 0$	$f(t) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

F

- Sampling distribution derived from the normal.
- $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ are random samples from n(μ_X, σ_X^2) and n(μ_Y, σ_Y^2) distributions.
- S_X^2 and S_Y^2 are the sample variances.
- $(S_X^2/\sigma_X^2)/(S_Y^2/\sigma_Y^2)$ has an F-distribution with $\nu_1 = n-1$ and $\nu_2 = m-1$ degrees of freedom.

Parameters	Probability Density Function	Expectation	Variance
$\nu_1 = 1, 2, \ldots$ $\nu_2 = 1, 2, \ldots$	$f(x) = \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2} \frac{x^{(\nu_1-2)/2}}{\left(1+\left(\frac{\nu_1}{\nu_2}\right)x\right)^{(\nu_1+\nu_2)/2}}$	$\frac{\nu_2}{\nu_2-2}, \nu_2 > 2$	$2\left(\frac{\nu_2}{\nu_2-2}\right)^2 \frac{\nu_1+\nu_2-2}{\nu_1(\nu_2-4)}, \nu_2 > 4$

Gamma

- $\{N(t) : t \geq 0\}$ is a Poisson process. $N(t)$ is number of events at or prior to time t.
- X_1 is time of first event, X_2 is elapsed time between first and second event etc.
- Time of nth event is random variable X with parameters (n, λ).

Parameters	Probability Density Function	Expectation	Variance
n $\lambda > 0$	$f(x) = \begin{cases} \lambda e^{-\lambda x} \frac{(\lambda x)^{n-1}}{(n-1)!} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$	$\frac{n}{\lambda}$	$\frac{n}{\lambda^2}$

Logistic

- Symmetric, unimodal distribution.
- Cumulative distribution function is the logistic distribution used in logistic regression.
- Heavier tails than the normal distribution.

Parameters	Probability Density Function	Expectation	Variance
$\mu =$ $\beta =$	$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1+\frac{x^2}{\nu}\right)^{(\nu+1)/2}}$	μ	$\frac{\pi^2\beta^2}{3}$

Lognormal

- X has a lognormal distribution if its logarithm is normally distributed.

Parameters	Probability Density Function	Expectation	Variance
$-\infty < \mu < \infty$ $\sigma > 0$	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \frac{e^{-(\log x - \mu)^2/(2\sigma^2)}}{x}$	$e^{\mu + (\sigma^2/2)}$	$e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$

Normal/Gaussian

- Approximation to binomial random variable as $n \to \infty$.

Parameters	Probability Density Function	Expectation	Variance
$-\infty < \mu < \infty$ $\sigma > 0$	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]$	μ	σ^2

Pareto

- Power-law distribution
- Shape parameter α controls exponent in power law.
- Scale parameter x_m controls lower bound of the distribution.

Parameters	Probability Density Function	Expectation	Variance
$\alpha > 0$ $x_m > 0$	$f(x) = \frac{\alpha x_m^\alpha}{x^{\alpha+1}}, \qquad x_m \leq x < \infty$	$\frac{\alpha x_m}{\alpha - 1}, \alpha > 1$	$\frac{\alpha x_m^2}{(\alpha-1)^2(\alpha-2)}, \alpha > 2$

Standard Normal

- Special case of normal distribution with $\mu = 0$, $\sigma^2 = 1$.

Parameters	Probability Density Function	Expectation	Variance
	$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$	0	1

T

- Sampling distribution derived from the normal.
- $X_1, \ldots, X_n$ is a random sample from a n(μ, σ^2) distribution.
- $\bar{X}$ and S are the sample mean and standard deviation.
- $(\bar{X} - \mu)/(S/\sqrt{n})$ has t-distribution with $\nu = n - 1$ degrees of freedom.

Parameters	Probability Density Function	Expectation	Variance
$\nu = 1, 2, \ldots$	$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{x^2}{\nu}\right)^{(\nu+1)/2}}$	$0, \quad \nu > 1$	$\frac{\nu}{\nu - 2}, \quad \nu > 2$

Uniform

- A point is randomly selected from an interval (a, b).
- Any two subintervals of equal length are equally likely to include the point.
- X, the value of the random point, is a **uniform random variable**.

Parameters	Probability Density Function	Expectation	Variance
$a < b$	$f(t) = \begin{cases} \frac{1}{b-a} & \text{if } a < t < b \\ 0 & \text{otherwise} \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$