

Noncommutative Chern-Weil theory and homotopy invariance of Hochschild and cyclic complexes

Goodwillie's theorem states that the periodic cyclic homology is invariant under nilpotent extensions. We discuss a special type of nilpotent extensions of unital algebras called row extensions. They appear in abundance, and are always H-unital but generically non-unital and noncommutative. For these special nilpotent extensions, we prove a stronger result: the homotopy invariance of Hochschild and cyclic complexes. A very specific type of a row extension appears naturally in the construction of the Chern-Galois character. If P is an algebra with a principal coaction, and B is its subalgebra of coaction invariants, then the Chern-Galois character factors through the row extension of B by the nilpotent ideal consisting of the invariant universal differential one-forms on P . When P is a principal comodule algebra, one can identify this ideal with the kernel of the multiplication map restricted to the algebra of the associated Ehresmann-Schauenburg quantum groupoid. For any principal coaction, all this leads to the Chern-Weil homomorphism defined on the space of cotraces.

Based on joint work with Tomasz Maszczyk.