Sub-Weyl bound for $GL(2)$ L-functions

Getting bounds for L-functions is a long-standing problem in number theory, both as an "easier" (but still fiendishly hard) substitute for the Riemann Hypothesis and as a type of result which in its own right has interesting applications to arithmetic, algebra, and geometry. One compelling reason to work on this problem is that it comes with a built-in way of measuring progress: there is always a "trivial" bound coming essentially from complex analysis, with no use of the arithmetic structure of the problem, known as the convexity bound (which is conductor$^{1/4}$). Beyond the convexity bound there are natural barriers that for some mysterious reason seem to crop up in all problems of this kind, called Burgess-type bounds (conductor$^{3/16}$) and the Weyl-type bounds (conductor$^{1/3}$). On $GL(2)$, a Weyl-type bound has been known for over 40 years due to Anton Good. In this talk we present joint work in progress with Roman Holowinsky, Ritabrata Munshi, and Prahlad Sharma where we for the first time cross the Weyl barrier on $GL(2)$.