Hecke algebras on homogeneous trees and relations with Hankel and Toeplitz matrices

A homogeneous tree of degree $q+1$ (a positive integer) is a connected graph, with no loops and with each vertex having exactly $q+1$ neighbours. The distance $d(x,y)$ between vertices x and y is the length of the uniquely defined geodesic connecting them. In particular there are $(q+1)q^{n-1}$ vertices at distance $n>0$ from a given one. In this talk we consider distance-dependent two-variable functions (kernels) $f(x,y)$, defined on pairs of vertices.

The Hecke algebra on homogeneous tree is a commutative algebra, spanned by particular kernels, defined on pairs of vertices (x,y) and indexed by non-negative integers $f_n(x,y)$. Each of these kernels depends on the distance between the vertices and vanishes if the distance is not equal to their index, otherwise it equals 1. We will show that the Hecke algebra is generated by f_1, which satisfies quadratic (Hecke) equation. Our main interest is in showing that the Hecke algebra is MASA (i.e. Maximal Abelian SubAlgebra) in some bigger algebra.

If $q>1$ the geometric trick of Y-turns on the tree will do the job. If $q=1$, corresponding to the tree of integers, Y-turns are not possible. We then introduce some additional (Banach space) structure and show that the Hecke algebra is not MASA, but its commutant decomposes as a direct sum of Hankel and Toeplitz (double-infinite) matrices.