Flat connections and toroidal algebras

Toroidal algebras are higher-(functional-)dimension analogues of affine algebras. Following Billig, we discuss representations of toroidal algebras using techniques in vertex operator algebras as well as certain extensions of them to superalgebras, and also to Hopf algebras which play the role of a Yangian of a toroidal algebra extended by divergence-free vector fields for some groups. We will also discuss a flat connection analogous to the trigonometric Casimir connection of a semisimple Lie algebra and explain how properties of the connection, including a tensor product structure and the quasimodularity connection, are related to properties of the Hopf algebra.