

Poisson geometry and the representation theory of cluster algebras

The relationship between Poisson geometry and cluster algebras was first studied by M. Gekhtman, M. Shapiro, and A. Vainshtein. Following their work, we study the global geometry picture of the affine Poisson varieties associated with a cluster algebra and its quantizations, root-of-unity quantum cluster algebras. In particular, we prove that the spectrum of the upper cluster algebra, endowed with the GSV Poisson structure, has a Zariski-open orbit of symplectic leaves and give an explicit description of it. Our result provides a generalization of the Richardson divisor of Schubert cells in flag varieties. Further, we describe the fully-Azumaya loci of the root-of-unity upper quantum cluster algebras, using the theory of Poisson orders. This classifies their irreducible representations of maximal dimension. This is joint work with Greg Muller, Kurt Trampel and Milen Yakimov.