Quantum symmetries of Frobenius algebras

We introduce the notions of "quantum support" and "quantum fundamental cycle" for a Frobenius algebra. We realize the Pareigis Hopf algebra, which encodes the monoidal structure of the category of complexes (via the Pareigis transform which is the identity on objects), as a universal quantum symmetry of the dual numbers algebra. We show that under the Pareigis transform the category of corresponding equivariant quasicoherent sheaves on the double point is equivalent to the category of complexes with square zero homotopies. In particular, the Pareigis transform of the algebra of dual numbers is the terminal object of the extended Hinich category of local pseudo-compact algebras.

We prove that the Pareigis transform of the Frobenius support of the algebra of dual numbers is a closed graded trace of dimension -1 on the terminal Hinich algebra, being a boundary of the Pareigis transform of the augmentation of dual numbers. This can be understood as a DGA model of the empty set with a homologically trivial (-1)-dimensional fundamental cycle. We also study symmetries of other truncated polynomial algebras and relate them to the representation theory of $SL(2)$, Hamiltonian moment maps, Fourier transforms, and the Springer resolution of the singularity of the nilpotent cone of $SL(2)$.