On the geometry of nilpotent orbits for classical simple Lie superalgebras

Many aspects of the representation theory of a Lie algebra and its associated algebraic group are governed by the geometry of their nilpotent cone. In this talk, we will introduce an analogue of the nilpotent cone $\mathcal{N}$ for Lie superalgebras and show that for a simple classical Lie superalgebra the number of nilpotent orbits is finite. We will also show that the commuting variety $\mathcal{X}$ described by Duflo and Serganova, which has applications in the study of the finite dimensional representation theory of Lie superalgebras, is contained in $\mathcal{N}$. Consequently, the finiteness result on $\mathcal{N}$ generalizes and extends the work on the commuting variety. For the general linear Lie superalgebra $\mathfrak{gl}(m|n)$, we will also discuss more detailed geometric results of $\mathcal{N}$. In particular, we compute the dimensions of $\mathcal{N}$ and the centralizer of a nilpotent orbit, describe the irreducible components of $\mathcal{N}$, and show that $\mathcal{N}$ is a complete intersection.

This is joint work with Daniel Nakano from the University of Georgia.