Springer correspondence for symmetric pairs and Hessenberg varieties

The Springer theory for reductive algebraic groups plays an important role in representation theory. It relates nilpotent orbits in the Lie algebra to irreducible representations of the Weyl group. We develop a Springer theory for symmetric pairs using Fourier transform and a nearby cycle sheaf construction. In this setting irreducible representations of Hecke algebras with parameter 1 and -1 enter the story. We explain its application to cohomology of Hessenberg varieties, examples of which include classical objects in algebraic geometry such as intersections of quadrics. The talk is based on joint work with K. Vilonen and partly with T.H. Chen and M. Grinberg.