

The parity of Lusztig's restriction functors and Green's formula

Our investigation in this talk is based on three important results. (1) Ringel introduced Hall algebra for representations of a quiver over finite fields and proved the elements corresponding to simple representations satisfy the quantum Serre relation. This gives a realization of the nilpotent part of quantum group if the quiver is of finite type. (2) Green found a homological formula for the representation category of the quiver and equipped Ringel's Hall algebra with a comultiplication. The generic form of the composition subalgebra of Hall algebra generated by simple representations realizes the nilpotent part of quantum group of any type. (3) Lusztig defined induction and restriction functors for the perverse sheaves on the variety of representations of the quiver which occur in the direct images of constant sheaves on flag varieties, and he found a formula between his induction and restriction functors which gives the comultiplication as algebra homomorphism for quantum group. In the present talk, we prove the formula holds for all semisimple complexes with Weil structure. This establishes the categorification of Green's formula.