Math 424 B / 574 B Wednesday, Oct 28

Autumn 2015

Midterm

Name: ______________________________

Student ID: ______________________________

1	15	
2	15	
Total	30+5=35	

- **Please read this.**
- You have 50 minutes.
- This is an open book exam. You can't collaborate (with anyone, either in class or electronically), but you're allowed any reading materials you like.
- Show all of your work and justify your answers unless instructed not to do so.
- If you want to use a theorem from the book, cite it by number; if you're using a homework problem, tell me which one.
- Not citing a previous result you're using could lose you points.
- Use the reverse side of each page for extra space and add paper of your own if you run out.

Problem 1. *Let* (X, d) *be a metric space,* $E \subseteq X$ *a subset. As usual, we denote the interior, closure and complement of* E *by* E°, $\overline{E}$ *and* E^c *respectively.*

Define the set ∂E *as follows:*

$$\partial E = \overline{E} \cap \overline{E^c}.$$

(a) *Show that* $\partial E = \overline{E} \setminus E^\circ$.

(b) *Show that the three sets* E°, ∂E *and* $(E^c)^\circ$ *are mutually disjoint (meaning no two of them intersect).*

(c) *Show that we have*

$$X = E^\circ \cup \partial E \cup (E^c)^\circ.$$

Solution.

Problem 2. *Let (X, d) be a metric space, and $E \subseteq X$ a subset.*

E is said to be **constrained** *if for every $\delta > 0$ there exist finitely many points $x_1, \cdots, x_n \in X$ such that*

$$E \subseteq \bigcup_{i=1}^{n} N_\delta(x_i).$$

(a) *Show that every compact subset of X is constrained.*
(b) *Show that every constrained subset of X is bounded.*
(c) *Show that every bounded subset of $\mathbb{R}$ is constrained.*
(d) **(Bonus: 2 extra points on top of the 35, but no partial credit)** *Show by example that a bounded subset of a metric space is not necessarily constrained.*

Solution.