Midterm

Name: ____________________________________________

Student ID: _______________________________________

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<td>30+5=35</td>
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- Please read this.
- You have 50 minutes.
- This is an open book exam. You can’t collaborate (with anyone, either in class or electronically), but you’re allowed any reading materials you like.
- Show all of your work and justify your answers unless instructed not to do so.
- If you want to use a theorem from the book, cite it by number; if you’re using a homework problem, tell me which one.
- Not citing a previous result you’re using could lose you points.
- Use the reverse side of each page for extra space and add paper of your own if you run out.
Problem 1. Let \((X, d)\) be a metric space, \(E \subseteq X\) a subset. As usual, we denote the interior, closure and complement of \(E\) by \(E^\circ\), \(\overline{E}\) and \(E^c\) respectively.

Define the set \(\partial E\) as follows:

\[ \partial E = \overline{E} \cap \overline{E^c}. \]

(a) Show that \(\partial E = \overline{E} \setminus E^\circ\).
(b) Show that the three sets \(E^\circ\), \(\partial E\) and \((E^c)^\circ\) are mutually disjoint (meaning no two of them intersect).
(c) Show that we have

\[ X = E^\circ \cup \partial E \cup (E^c)^\circ. \]

Solution.
Problem 2. Let $(X, d)$ be a metric space, and $E \subseteq X$ a subset.

$E$ is said to be constrained if for every $\delta > 0$ there exist finitely many points $x_1, \ldots, x_n \in X$ such that

$$E \subseteq \bigcup_{i=1}^{n} N_{\delta}(x_i).$$

(a) Show that every compact subset of $X$ is constrained.
(b) Show that every constrained subset of $X$ is bounded.
(c) Show that every bounded subset of $\mathbb{R}$ is constrained.
(d) (Bonus: 2 extra points on top of the 35, but no partial credit) Show by example that a bounded subset of a metric space is not necessarily constrained.

Solution.