

Homework 7

Problems 6, 8 and 11 from the end of Chapter 4, on page 99 and the two additional problems below (the first of which is a complement to Problem 11).

E1. Give an example of a continuous function $f : (0, 1) \rightarrow \mathbb{R}$ and a Cauchy sequence $(x_n)_n$ in $(0, 1)$ such that $(f(x_n))_n$ is not Cauchy in \mathbb{R} .

E2. Show that a metric space (X, d) is disconnected if and only if there exists a continuous function $f : X \rightarrow \mathbb{R}$ whose range $f(X)$ is the two-element set $\{0, 1\}$.

For Problem 6, assume E is a subset of \mathbb{R} (the statement is a bit unclear on this point). As a hint, try looking at the function $F : E \rightarrow \mathbb{R}^2$ defined by

$$F(x) = (x, f(x))$$

whose range is exactly the graph of $f : E \rightarrow \mathbb{R}$.