Math 424 B / 574 B Due Wednesday, Oct 21
Autumn 2015

Homework 3

Problems 6, 7, 9, 12, 15, 23 from the end of Chapter 2.

I will now follow up with some hints for some of these.

Problem 6. For the last question (on whether E and E' always have the same limit points): You've seen (repeatedly) an example in $\mathbb{R}$ where E' is a singleton (that is, a single-element set) that is not contained in E. Will that work?

Problem 7. Regarding the example you have to cook up for part (b): What if the sets A_n are singletons in $\mathbb{R}$? Can you come up with an example such that these single points, as n grows large, approach some point that's different from each of them?

Problem 9. For parts (e) and (f), it might be helpful to first come up with some set E whose interior E° is empty. Will that do for (f)?

As for (e), maybe you can arrange for such an E that in addition is dense in whatever ambient metric space you're in.

Problem 15. It's enough to come up with examples that violate the corollary when you replace 'compact' with 'closed' or 'bounded'.

To make your life easy, I suggest you seek examples in $\mathbb{R}$.

To see that 'closed' won't work: You know from Theorem 2.41 that closed + bounded automatically means compact, so you'll want your examples to be *un*bounded. What if you take them to be *really* long intervals?

On the other hand, you have to also show that 'bounded' won't do instead of 'compact'. Again, because Theorem 2.41 says that closed and bounded implies compact, your examples had better *not* be closed.

Maybe you can come up with a shrinking sequence of open intervals that does the trick (meaning it has empty intersection).