**Math 424 B / 574 B** Due Wednesday, Oct 14
Autumn 2015

# Homework 2

Problems 2, 5 from the end of Chapter 2, plus the following (I'll name them E+number, for 'extra').

First, I'll need the following definition to complement Definition 2.9 from your textbook.

**Definition 1.** Let $A$ and $B$ be two sets. The *set difference* $A \setminus B$ (sometimes also denoted by $A - B$) is

$$\{a \in A : \ a \notin B\}.$$

In other words, it is the set of elements of $A$ that are not elements of $B$. ♦

**E1.** Prove that for any two sets $A$ and $B$ we have

$$(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A). \tag{1}$$

The set (1) is also known as the *symmetric difference* of the sets $A$ and $B$, and it's sometimes denoted by $A \Delta B$.

**E2.** Let $f : X \to Y$ be a function between two sets. Show that for subsets $A, B \subseteq Y$ we have

$$f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$$

and

$$f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B).$$

**E3.** Let $f : X \to Y$ be a map from $X$ to $Y$

(a) Show that for subsets $C, D \subseteq X$ we have $f(C \cup D) = f(C) \cup f(D)$.
(b) For $C$ and $D$ as before, show that we also have $f(C \cap D) \subseteq f(C) \cap f(D)$.
(c) Take $X = Y = \mathbb{R}$ and $f(x) = x^2$. Find two subsets $C$ and $D$ of $\mathbb{R}$ such that $f(C \cap D) \neq f(C) \cap f(D)$.
(d) Going back to the general case of an arbitrary map $f : X \to Y$, show that the statement

$$f(C \cap D) = f(C) \cap f(D) \text{ for all possible choices of } C, D \subseteq X$$

is equivalent to $f$ being one-to-one.