Bonus homework

These questions are meant to get you to explore the notion of perfect set a bit, getting you some additional points towards your course grade along the way.

There is no penalty if you do not turn them in, and no partial credit: each of the three questions, if answered correctly with correct justification, is worth 0.5% extra credit.

1. Does there exist a non-empty perfect subset of $\mathbb{R}$ that contains no rational numbers?

2. Same as above, with 'irrational' instead of 'rational'.

3. Same as before, with 'algebraic' instead of '(ir)rational'.