IE675 Game Theory

Lecture Note Set 1

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1 INTRODUCTION

1.1 What is game theory?

Game theory is the study of problems of conflict and cooperation among independent decision-makers.

Game theory deals with *games of strategy* rather than *games of chance.*

The ingredients of a game theory problem include

- players (decision-makers)
- choices (feasible actions)
- payoffs (benefits, prizes, or awards)
- preferences to payoffs (objectives)

We need to know when one choice is better than another for a particular player.

1.2 Classification of game theory problems

Problems in game theory can be classified in a number of ways.

1.2.1 Static vs. dynamic games

In dynamic games, the order of decisions are important.

Question 1.1. Is it ever *really* possible to implement static decision-making in practice?

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1.2.2 Cooperative vs. non-cooperative

In a non-cooperative game, each player pursues his/her own interests. In a cooperative games, players are allowed to form coalitions and combine their decision-making problems.

	Non-cooperative	*Cooperative*
Static	Math programming *and* Non-cooperative Game Theory	Cooperative Game Theory
Dynamic	Control Theory	Cooperative Dynamic Games

Note 1.1. This area of study is distinct from multi-criteria decision making.

Flow of information is an important element in game theory problems, but it is sometimes explicitly missing.

- noisy information
- deception

1.2.3 Related areas

- differential games
- optimal control theory
- mathematical economics

1.2.4 Application areas

- corporate decision making
- defense strategy
- market modelling
- public policy analysis
- environmental systems
- distributed computing
- telecommunications networks

1.2.5 Theory vs. simulation

The mathematical theory of games provides the fundamental laws and problem structure. Games can also be simulated to assess complex economic systems.

1.3 Solution concepts

The notion of a "solution" is more tenuous in game theory than in other fields.

Definition 1.1. *A* **solution** *is a systematic description of the outcomes that may emerge from the decision problem.*

- optimality (for whom??)
- feasibility
- equilibria

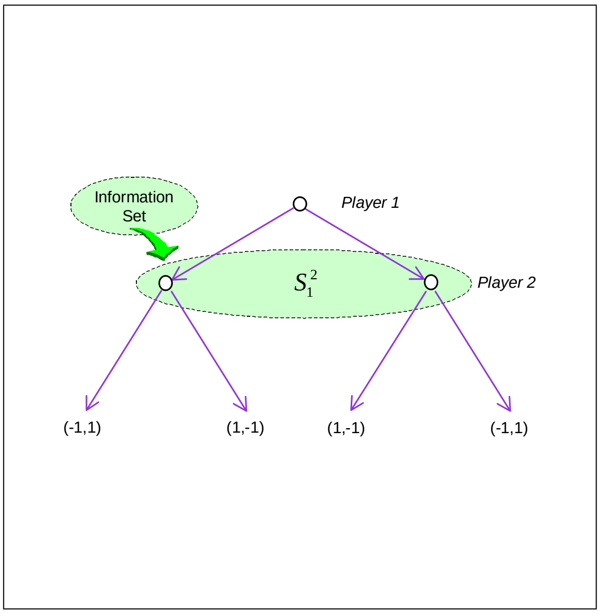
1.4 Games in extensive form

1.4.1 Example: Matching Pennies

- **Player 1:** Choose H or T
- **Player 2:** Choose H or T (not knowing Player 1's choice)

- If the coins are alike, Player 2 wins 1 cent from Player 1
- If the coins are different, Player 1 wins 1 cent from Player 2

Written in extensive form, the game appears as follows

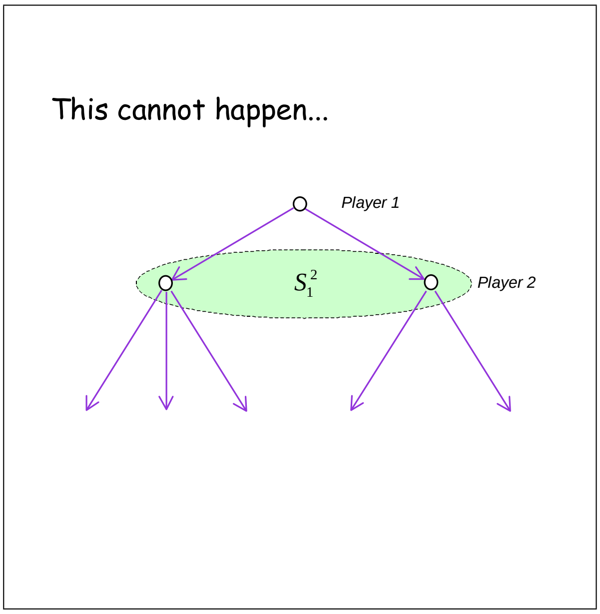


In order to deal with the issue of Player 2's knowledge about the game, we introduce the concept of an *information set*. When the game's progress reaches Player 2's time to move, Player 2 is supposed to know Player 1's choice. The set of nodes, S_1^2, is an information set for Player 2.

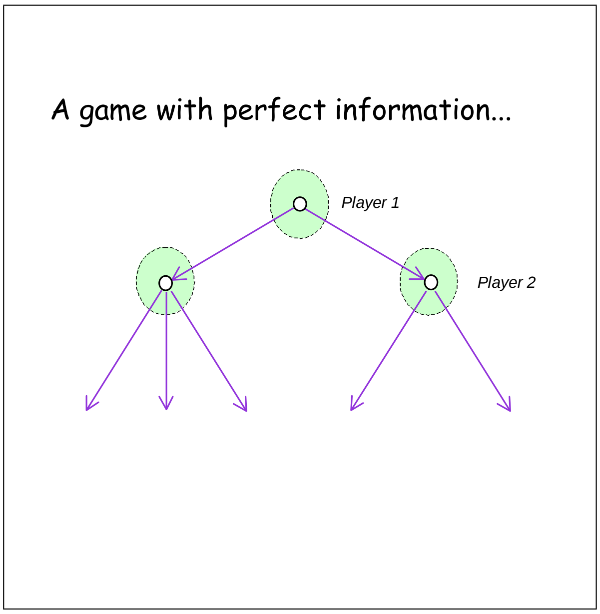
A player only knows the possible options emanating from an information. A player does not know which node within the information set is the actual node at which the progress of play resides.

There are some obvious rules about information sets that we will formally describe later.

For example, the following cannot occur. . .



Definition 1.2. *A player is said to have* **perfect information** *if all of his/her information sets are singletons.*



Definition 1.3. *A* **strategy** *for Player i is a function which assigns to each of Player i's information sets, one of the branches which follows a representative node from that set.*

Example 1.1. A strategy Ψ which has $\Psi(S_1^2) = \mathrm{H}$ would tell Player 2 to select *Heads* when he/she is in information set S_1^2.

1.5 Games in strategic form

1.5.1 Example: Matching Pennies

Consider the same game as above and the following matrix of payoffs:

		Player 2	
		H	T
Player 1	H	(-1,1)	(1,-1)
	T	(1,-1)	(-1,1)

The rows represent the strategies of Player 1. The columns represent the strategies of Player 2.

Distinguish *actions* from *strategies*.

The matching pennies game is an example of a *non-cooperative game*.

1.6 Cooperative games

Cooperative games allow players to form coalitions to share decisions, information and payoffs.

For example, if we have player set

$$N = \{1, 2, 3, 4, 5, 6\}$$

A possible coalition structure would be

$$\{\{1, 2, 3\}, \{4, 6\}, \{5\}\}$$

Often these games are described in *characteristic function form.* A *characteristic function* is a mapping

$$v : 2^N \to \mathbb{R}$$

and $v(S)$ (where $S \subseteq N$) is the "worth" of the coalition S.

1.7 Dynamic and multistage games

Dynamic games can be cooperative or non-cooperative in form.

One class of cooperative dynamic games are hierarchical (organizational) games.

Consider a hierarchy of players, for example,

President
Vice-President
⋮
Workers

Each level has control over a subset of all decision variables. Each may have an objective function that depends on the decision variables of other levels. Suppose that the top level makes his/her decision first, and then passes that information to the next level. The next level then makes his/her decision, and the play progresses down through the hierarchy.

The key ingredient in these problems is preemption, i.e., the "friction of space and time."

Even when everything is linear, such systems can produce *stable, inadmissible* solutions (Chew).

stable: No one wants to unilaterally move away from the given solution point.

inadmissible: There are feasible solution points that produce better payoffs for all players than the given solution point.